
Effects of viscous dissipation on unsteady MHD free convective flow with thermophoresis past a radiate inclined permeable plate

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Abstract

An analysis is carried out to investigate the effects of variable chemical reaction, thermophoresis, temperature-dependent viscosity and thermal radiation on an unsteady MHD free convective heat and mass transfer flow of a viscous, incompressible, electrically conducting fluid past an impulsively started infinite inclined porous plate by taking into account the viscous dissipation effects. The governing nonlinear partial differential equations are transformed into a system of ordinary differential equations, which are solved numerically by using implicit finite difference scheme with shooting method. Numerical results for the non-dimensional velocity, temperature and concentration profiles as well as the local skin-friction coefficient, the local Nusselt number and the local Stanton number are presented for different physical parameters. The results show that variable viscosity significantly increases viscous drag and rate of heat transfer. The results also show that higher order chemical reaction induces the concentration of the particles for a destructive reaction and reduces for a generative reaction.

Keywords: Variable viscosity; chemical reaction; thermophoresis; MHD; thermal radiation; finite difference scheme; viscous dissipation

1. Introduction

Dissipation is the process of converting mechanical energy of downward-flowing water into thermal and acoustical energy. Various devices are designed in streambeds to reduce the kinetic energy of flowing waters, reducing their erosive potential on banks and river bottoms. In nature, the presence of pure air or water is not possible. Some foreign mass may be present either naturally or mixed with the air or water. The presence of a foreign mass in air or water causes some kind of chemical reaction. The study of such type of chemical reaction processes is useful for improving a number of chemical technologies, such as food processing and polymer production. This study is more important in industries such as hot rolling, melt spinning, extrusion, glass fiber production, wire drawing, and manufacture of plastic and rubber sheets, polymer sheet and filaments, etc. Chamber and Young (1958) analyzed the problem of first order chemical reactions in the neighborhood of a flat plate for destructive and generative reactions. Anjalidevi and Kandasamy (2000) analyzed the effect of a chemical reaction on the flow in the presence of heat transfer and magnetic field. Muthucumaraswamy and Ganesan (2001) studied

the effect of a chemical reaction on an unsteady flow past an impulsively started vertical plate which is subjected to uniform mass flux and in the presence of heat transfer. Alam et al. (2007) investigated the effects of thermophoresis and first order chemical reaction on unsteady hydromagnetic free convection and mass transfer flow past an impulsively started infinite inclined porous plate in the presences of heat generation/absorption.

When a temperature gradient is established in gas, small particles suspended in the gas migrate in the direction of decreasing temperature. This phenomenon, called thermophoresis, occurs because gas molecules colliding on one side of a particle have different average velocities from those on the other side due to the temperature gradient. Hence, when a cold wall is placed in the hot particle-laden gas flow, the thermophoretic force may cause particles to be deposited on it. The magnitude of this force depends on gas and particle properties as well as on the temperature gradient. The thermophoretic deposition plays an important role in a variety of applications such as the production of ceramic powders in high temperature aerosol flow reactors; the production of optical fiber is performed by the modified chemical vapor deposition (MCVD) process and in polymer separation. Thermophoresis is considered to be important for particles of 10 m in radius and

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temperature gradient of the order of 5K/mm. Walker et al. (1979) calculated the deposition efficiency of small particles due to thermophoresis in a laminar tube flow. The effect of wall suction and thermophoresis on aerosol-particle deposition from a laminar boundary layer on a flat plate was studied by Mills et al. (1984). Ye et al. (1991) analyzed the thermophoretic effect of particle deposition on a free standing semiconductor wafer in a clean room. Recently, Alam and Rahman (2006) investigated numerically the effect of thermophoresis on surface deposition flux on hydromagnetic free convective heat and mass transfer flow along a semi-infinite permeable inclined flat plate considering heat generation. Their results show that thermophoresis increases surface mass flux significantly.

The study of magneto hydrodynamic flow with viscous and Joules dissipation has many important industrial, technological and geothermal applications such as high temperature plasmas cooling of nuclear reactors, liquid metal fluids, MHD accelerators and power generation systems. Heat transfer and fluid flow due to free convection in the presence of magnetic field and internal heat generation find useful applications in different branches of science and technology such as nuclear science, fire engineering combustion modeling, geophysical etc.

The previous studies, which dealt with thermophoresis, neglected the effect of thermal radiation. But in fact, there are various kinds of high temperature systems such as a heat exchanger and an internal combustor, in which the radiation may not be negligible in comparison with the conductive and convective heat transfer mode. Usually, the gas like CO₂ and H₂ O generated during combustion of a hydrocarbon fuel and the particles such as soot and coal suspended in a hot gas flow absorb, emit and scatter the radiation. In light of this importance, the radiation effect on the thermophoresis of particles was analyzed for a gas particle two phase laminar flow by Sohn et al. (2002). Akbar and Ghiaasiaan (2005) studied numerically the combined effects of radiation heat transfer and thermophoresis on the transport of monodisperse, as well as polydisperse soot particles. Alam et al. (2008) studied numerically the thermal radiation interaction of thermophoresis on free-forced convective heat and mass transfer flow along a semi-infinite permeable inclined flat plate. Their results show that interaction of thermal radiation with thermophoresis depositing surface mass flux is significant. Cortell (2008) investigated theoretically the effects of viscous dissipation as well as radiation on the thermal boundary layer flows over nonlinearly stretching sheets considering prescribed surface temperature and heat flux.

Most of the above studies are based on the constant physical properties of the ambient fluid. However, it is known that these properties may change with temperature, especially fluid viscosity. To accurately predict the flow behavior and heat transfer rate, it is necessary to take this variation of viscosity into account. Molla and Hossain (2006) investigated the effects of chemical reaction, heat and mass diffusion in natural convection flow from, and isothermal sphere with temperature-dependent viscosity. Alam, et al. (2009) studied the effects of higher order chemical reaction and thermophoresis on an unsteady MHD free convective heat and mass transfer flow past and impulsively started infinite inclined porous plate in the presence of thermal radiation with temperature-dependent viscosity. Nadeem et al. (2012) discussed MHD Boundary Layer Flow of a Nano Fluid past a Porous Shrinking Sheet with Thermal Radiation. Nadeem et al. (2013) analyzed the MHD three-dimensional Casson fluid flow past a porous linearly stretching sheet. Nadeem et al. (2013) studied MHD boundary layer flow of a Maxwell fluid past a stretching sheet in the presence of nanoparticles.

Therefore the objective of the present paper is to investigate the effect of higher order chemical reaction and thermophoresis on an unsteady MHD free convective heat and mass transfer flow past an impulsively started infinite inclined porous plate in the presence of thermal radiation with temperature-dependent viscosity, by taking into account the viscous dissipation.

2. Formulation of the problem

We consider an unsteady MHD free convective heat and mass transfer flow of an electrically conducting (for example, plasmas, liquid metals, salt water, air etc) incompressible viscous fluid past an infinite inclined porous flat plate with an acute angle α to the vertical. The flow is assumed to be in the x -direction, which is taken along the inclined plate and y -axis normal to it. A magnetic field of uniform strength B_0 is introduced normal to the direction of the flow. Initially ($t=0$) the plate and the fluid are at rest. But for time $t > 0$, the plate starts moving impulsively in its own plane with a velocity U_0 , and its temperature is raised to T_w which is higher than the ambient temperature T_∞ . The species concentration at the surface is maintained uniform at C_w , which is taken to be zero and that of the ambient fluid is assumed to be C_∞ . Fluid suction or injection is imposed at the plate surface. In addition, the effect of thermophoresis is taken into account as it helps in understanding mass deposition on surface. We further assume that (i) the magnetic Reynolds number is assumed to be small so that the induced magnetic field is

negligible in comparison to the applied magnetic field, (ii) due to the boundary layer behavior the temperature gradient in the y -direction is much larger than in the x -direction and hence only the thermophoretic velocity component which is normal to the surface is of importance, (iii) the fluid is considered to be gray; absorbing-emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the radioactive heat flux in the x -direction, (iv) the fluid property variation with temperature is limited to viscosity, (v) there exists a homogeneous n -th order chemical reaction between the fluid and species concentration, (vi) joule heating terms are neglected for low speed flows and, (vii) the level of species concentration is very low so that the heat generated during chemical reaction can be neglected.

Since the plate is of infinite extent, all derivatives are supposed to be zero and hence under the usual Boussinesq and boundary-layer approximation, the governing equations for this problem can be written as follows:

$$\frac{\partial u}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = \frac{1}{\rho_\infty} \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) + g\beta(T - T_\infty) \cos \alpha - \frac{\sigma B_0^2}{\rho_\infty} u \quad (2)$$

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \frac{\lambda_g}{\rho_\infty c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho_\infty c_p} \frac{\partial q_r}{\partial y} + \mu \left(\frac{\partial u}{\partial y} \right)^2 \quad (3)$$

$$\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - \frac{\partial}{\partial y} (V_T C) - K_l C^n \quad (4)$$

Where u, v are the velocity components in the x and y directions respectively, t is the time, μ is the fluid viscosity, g is the acceleration due to gravity, σ is the electrical conductivity, ρ_∞ is the ambient density, β is the volumetric coefficient of thermal expansion, T, T_w and T_∞ are the temperature of the fluid inside the thermal boundary layer, the plate temperature and the fluid temperature in the free stream, respectively, while C, C_w and C_∞ are the corresponding concentrations, B_0 is the magnetic induction, λ_g is the thermal conductivity of the fluid, c_p is the specific heat at constant pressure, q_r is the radiative heat flux in the y -direction, D is the molecular diffusivity of the species concentration, V_T is the thermophoretic velocity, K_l is the chemical reaction parameter and n is the order of chemical reaction.

The initial and boundary conditions for the above problem are:

$$\begin{aligned} \text{For } t \leq 0; u = v = 0, T = T_\infty, C = C_\infty, \text{ for all } y \\ u = U_0, v = \pm v(t), T = T_w, C = C_w = 0 \text{ at } y=0 \\ \text{For } t > 0: u = 0, T = T_\infty, C = C_\infty = 0 \text{ as } y \rightarrow \infty. \end{aligned} \quad (5)$$

Where $v(t)$ is the time dependent suction/injection velocity at the porous plate. The radiative heat flux under Rosseland approximation has the form

$$q_r = -\frac{4\sigma_1}{3k_1} \frac{\partial T^4}{\partial y} \quad (6)$$

Where σ_1 is the Stefan-Boltzmann constant and k_1 is the mean absorption coefficient.

We assume that the temperature difference within the flow is sufficiently small such that T^4 may be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (7)$$

Using (6) and (7) in equation (3) we have

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \frac{\lambda_g}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma_1 T_\infty^3}{3\rho c_p k_1} \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 \quad (8)$$

In equation (4), the thermophoretic velocity V_T is given by

$$V_T = -k\nu \frac{\nabla T}{T_0} = -\frac{k\nu}{T_0} \frac{\partial T}{\partial y} \quad (9)$$

Where T_0 is some reference temperature, the value of $k\nu$ represents the thermophoretic diffusivity, and k is the thermophoretic coefficient which ranges in value from 0.2 - 1.2.

$$k = \frac{2C_s (\lambda_g / \lambda_p + C_t Kn) [1 + Kn (C_1 + C_2 e^{-C_3 / Kn})]}{(1 + 3C_m Kn)(1 + 2\lambda_g / \lambda_p + 2C_t Kn)} \quad (10)$$

Where $C_1, C_2, C_3, C_m, C_s, C_t$ are constants, λ_g and λ_p are the thermal conductivities of the fluid and diffused particles, respectively and Kn is the Knudsen number.

A thermophoretic parameter τ can be defined as follows:

$$\tau = -\frac{k(T_w - T_\infty)}{T_0} \quad (11)$$

Typical values of τ are 0.01, 0.05 and 0.1 corresponding to approximate values of $-k(T_w - T_\infty)$ equal to 3, 15 and 30 K for a reference temperature of $T_0 = 300\text{K}$. A viscosity dependence on the temperature T of the form

$$\mu = \frac{\mu_\infty}{[1 + \gamma(T - T_\infty)]} \quad (12)$$

So that viscosity is an inverse linear function of temperature T .

Equation (12) can be rewritten as

$$\frac{1}{\mu} = A(T - T_r) \quad (13)$$

Where $A = \frac{\gamma}{\mu_\infty}$ and $T_r = T_\infty - \frac{1}{\gamma}$.

In the above relations, both A and T_r are constants and their values depend on the reference state and γ , a thermal property of the fluid. In general, $A > 0$ for liquids, and $A < 0$ for gases. Typical values of γ and A for air are $\gamma = 0.026240$ and $A = -123.2$

To obtain local similarity solution at the time of the problem under consideration, we introduce a time dependent length scale δ as

$$\delta = \delta(t) \quad (14)$$

In terms of this length scale, a convenient solution of the equation (1) is considered to be in the following form:

$$v = v(t) = -v_0 \frac{v}{\delta} \quad (15)$$

Where v_0 is the suction/ injection parameter.

We now introduce the following dimensionless variables:

$$\eta = \frac{y}{\delta}, \quad u = U_0 f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (16)$$

$$\phi(\eta) = \frac{C}{C_\infty}$$

The dimensionless temperature θ can also be written as:

$$\theta = \frac{T - T_r}{T_w - T_\infty} + \theta_r \quad (17)$$

$$\text{Where } \theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = -\frac{1}{\gamma(T_w - T_\infty)} = \text{const} \quad (18)$$

and its value is determined by the viscosity/temperature characteristics of the fluid under consideration and the operating temperature difference $\Delta T = T_w - T_\infty$

Then introducing the relations (9), (12), (13), (15), (16) and (18) into the equations (2),(3) and (4), respectively, we obtain the following ordinary differential equations:

$$f'' + \eta \left(\frac{\delta d\delta}{v dt} \right) \left(\frac{\theta_r - \theta}{\theta_r} \right) f' + v_0 \left(\frac{\theta_r - \theta}{\theta_r} \right) f' + \left(\frac{\theta}{\theta_r - \theta} \right) f \quad (19)$$

$$+ Gr \left(\frac{\theta_r - \theta}{\theta_r} \right) \theta \cos \alpha - M \left(\frac{\theta_r - \theta}{\theta_r} \right) f = 0$$

$$- \eta \left(\frac{\delta d\delta}{v dt} \right) \theta' - v_0 \theta' = \left(\frac{3R+4}{3RPr} \right) \theta' + \left(\frac{\theta_r}{\theta_r - \theta} \right) Ec (f')^2 \quad (20)$$

$$- \eta \left(\frac{\delta d\delta}{v dt} \right) \phi' - v_0 \phi' = \frac{1}{Sc} \phi'' - \tau (\phi \theta'' + \phi' \theta') - K \phi'' \quad (21)$$

Where $Pr = \frac{\rho_\infty c_p v}{\lambda_g}$ is the Prandtl number,

$Sc = \frac{v}{D}$ is the Schmidt number, $M = \frac{\sigma B_0^2 \delta^2}{v \rho_\infty}$

is the local magnetic field parameter,

$Gr = \frac{g \beta (T_w - T_\infty) \delta^2}{v U_0}$ is the local Grashof

number, $R = \frac{\lambda_g k_1}{4 \sigma_1 T_\infty^3}$ is the radiation parameter,

$K = \frac{K_1 \delta^2}{v}$ is the local chemical reaction

parameter and is the Eckert number.

The corresponding boundary conditions for $t > 0$ are obtained as:

$$f = 1, \quad \theta = 1, \quad \phi = 0 \quad \text{at } \eta = 0$$

$$f = 0, \quad \theta = 0, \quad \phi = 1 \quad \text{at } \eta \rightarrow \infty \quad (22)$$

Now the equations (19)-(21) are locally similar except the term $\left(\frac{\delta d\delta}{v dt} \right)$ where t appears

explicitly. Thus the local similarity condition

requires that the $\left(\frac{\delta}{\nu} \frac{d\delta}{dt}\right)$ term in the equations

(19)-(21) must be constant quantity.

Hence, following the works of Hasimoto (1956), Sattar and Hossain (1992), Alam et al. (2006) Rahman and Sattar (2007) one can try a class of solutions of the equations (19)-(21) by assuming that

$$\left(\frac{\delta}{\nu} \frac{d\delta}{dt}\right) = \lambda \quad (\text{a constant}) \quad (23)$$

Integrating (23) we have

$$\delta = \sqrt{2\lambda\nu t}. \quad (24)$$

Where the constant of integration is determined through the condition that $\delta = 0$ when $t = 0$. We have considered the problem for small time. In this case normal velocity in (15) will be large i.e., suction will be large, which can be applied to increase the lift of the airfoils. From (23), choosing $\lambda = 2$, the length scale $\delta(t) = 2\sqrt{\nu t}$ which exactly corresponds to the usual scaling factor for various unsteady boundary layer flows. Since it is a scaling factor as well as a similarity parameter, any value of λ in (23) would not change the nature of the solution except that the scale would be different.

Now introducing (23) (with $\lambda = 2$) in the equations (19)-(21) respectively, we obtain the following dimensionless non-linear ordinary differential equations which are locally similar in time but not explicitly time dependent.

$$f'' + (2\eta + \nu_0) \left(\frac{\theta_r - \theta}{\theta_r}\right) f' + \left(\frac{\theta'}{\theta_r - \theta}\right) f' \quad (25)$$

$$+ Gr \left(\frac{\theta_r - \theta}{\theta_r}\right) \theta \cos \alpha - M \left(\frac{\theta_r - \theta}{\theta_r}\right) f = 0$$

$$\theta' + (2\eta + \nu_0) \left(\frac{3RPr}{3R+4}\right) \theta' + \left(\frac{3RPr}{3R+4}\right) \left(\frac{\theta_r}{\theta_r - \theta}\right) Ec(f')^2 = 0 \quad (26)$$

$$\phi'' + Sc(2\eta + \nu_0) \phi' - \tau Sc(\phi\theta'' + \phi'\theta') - ScK\phi^n = 0 \quad (27)$$

Where primes denote differentiation with respect to similarity variable η .

Local skin-friction coefficient, local Nusselt number and local Stanton number

The physical quantities of most interest for the present problem are the local skin-friction coefficient (dimensionless wall shear stress), the local Nusselt number (rate of heat transfer) and the local Stanton number (rate of transfer of species

concentration) which are defined respectively by the following relations:

$$C_f = \frac{\tau_w}{\rho U_0^2}, Nu = \frac{\delta q_w}{\lambda_g (T_w - T_\infty)}, \quad (28)$$

$$St = -\frac{J_s}{U_0 C_\infty}$$

Now the wall shear stress on the surface τ_w , rate of heat transfer q_w and rate of transfer of species concentration J_s are given by

$$\tau_w = \left[\mu \frac{\partial u}{\partial y} \right]_{y=0} \quad (29)$$

$$q_w = -\lambda_g \left(\frac{\partial T}{\partial y}\right)_{y=0} - \frac{4\sigma}{3k_1} \left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (29.1)$$

$$J_s = -D \left(\frac{\partial C}{\partial y}\right)_{y=0} \quad (29.2)$$

Now using equation (7) we have from (29.1)

$$q_w = -\lambda_g \left(\frac{\partial T}{\partial y}\right)_{y=0} - \frac{16\sigma_1 T_\infty^3}{3k_1} \left(\frac{\partial T}{\partial y}\right)_{y=0}$$

$$= -\frac{\lambda_g \Delta T}{\delta} \theta'(0) - \frac{16\sigma_1 T_\infty^3 \Delta T}{3k_1 \delta} \theta'(0) \quad (30)$$

$$= -\frac{\lambda_g \Delta T}{\delta} \left(1 + \frac{4}{3R}\right) \theta'(0)$$

Using (12), (16), (29), and (30) the quantities of (28) can be written as follows:

$$C_f Re = \left(\frac{\theta_r}{\theta_r - 1}\right) f'(0), Nu = -\left(1 + \frac{4}{3R}\right) \theta'(0) \quad (31)$$

$$St Sc Re = \phi'(0)$$

Where $Re = \frac{U_0 \delta}{\nu}$ is the local Reynolds number

3. Method of solution

The systems of non-linear and local similar ordinary differential Equations (25)-(27) together with the boundary conditions equation (22) have been solved numerically by using implicit finite difference scheme along with the Gauss seidel

iteration method. By adopting this numerical technique a computer C-programming was used to get the numerical values. In all the computations the step size $\Delta \eta=0.01$ was selected, that satisfied a convergence criterion of 10^{-5} in almost all of the different phases mentioned above. The value of η_{∞} was found to each iteration loop by the statement $\eta_{\infty} = \eta_{\infty} + \Delta \eta$ The maximum value of η_{∞} for each group of parameters has been obtained when the value of the unknown boundary conditions at $\eta = 0$ does not change in the successful loop with an error less than 10^{-5} .

In various groups the parameters v_0, θ_r, μ etc were considered in different phases. In all of the computations the step size $\Delta \eta = 0.001$ was selected, which satisfied a convergence criteria of 10^{-5} . Where numerical simulation, step size

$\Delta \eta = 0.01, 0.005$ were all checked and values of $f''(0)$ and $\theta'(0)$ were found in each case correct up to five decimal places. Hence, the scheme used in this paper is stable and accurate.

4. Results and Discussion

The numerical values are computed for velocity, temperature and concentration profiles for different parameters. Figs. 1-4 are plotted for the velocity profiles for different flow parameters θ_r, M, α, R . From Fig. 1 it is observed that the velocity profiles increase with the increase of θ_r . The effect of magnetic field parameter M is displayed in Fig. 2. The effect of magnetic parameter on velocity profiles is to decelerate the velocity profiles is observed in Fig. 2.

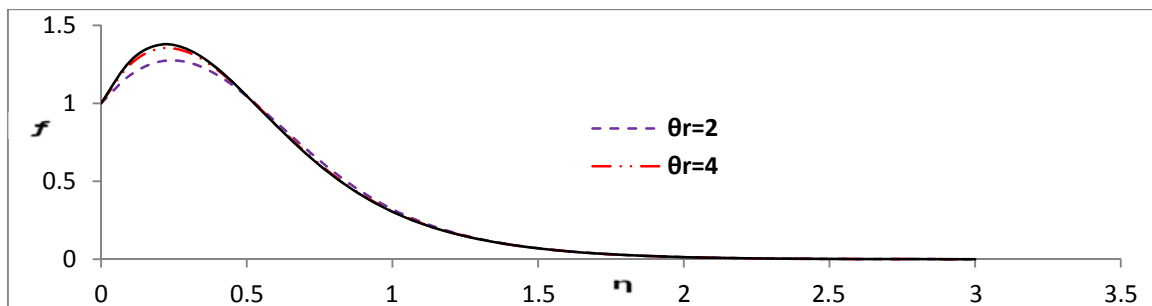


Fig. 1. Velocity profiles for different values of θ_r

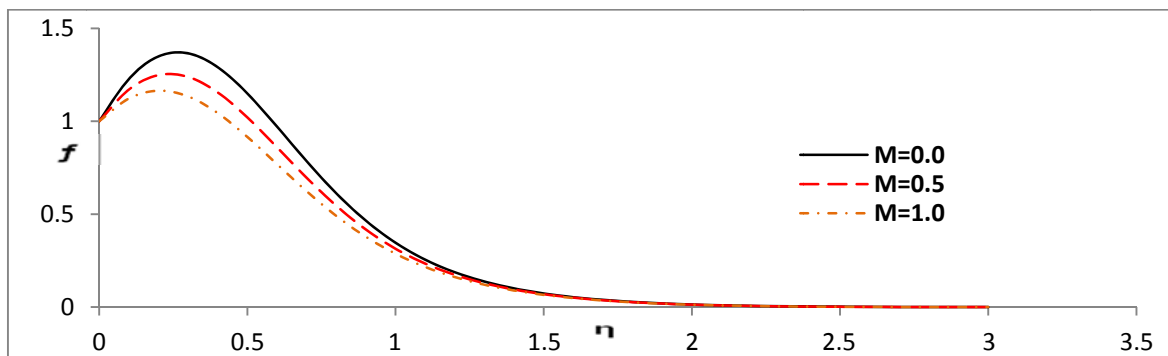


Fig. 2. Velocity profiles for different values of M

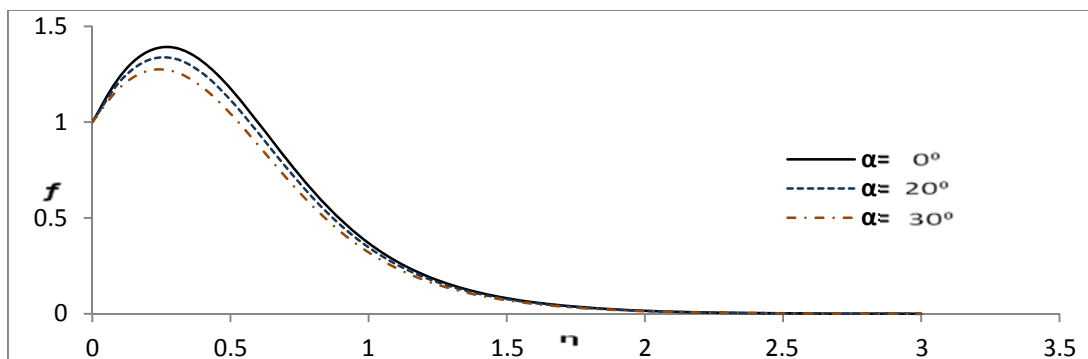


Fig. 3. Velocity profiles for different values of α

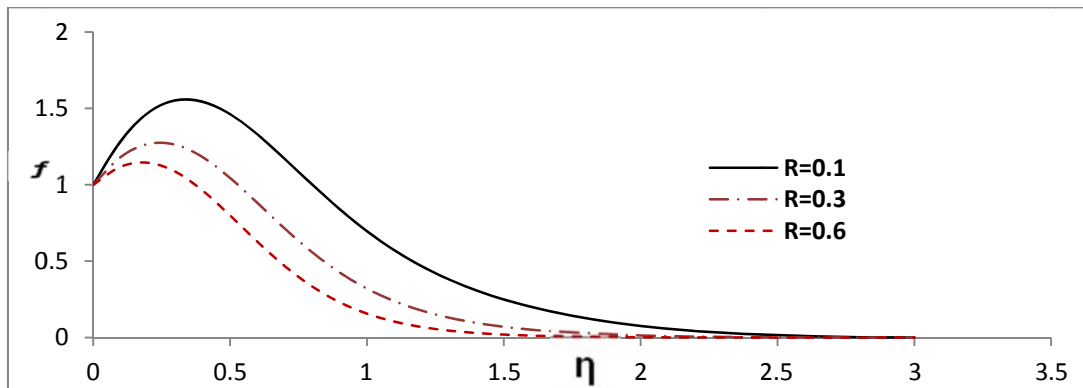


Fig. 4. Velocity profiles for different values of R

Figure 3 shows that the velocity profile f decreases by increasing the angle of inclination α . The fact is that the angle of inclination increases the effect of the Buoyancy force due to thermal diffusion decreases by a factor of $\cos \alpha$, consequently the driving force to the fluid decreases as a result of velocity profile decrease. It is noted from Fig. 4 that the velocity decreases with the effect of radiation parameters R.

The temperature profiles are shown in Figs. 5 and 6 for different values of R and Ec . It is seen from this Fig. that the temperature profiles decrease with the increase of radiation parameter. The effect of Eckert number Ec leads to increase the temperature profiles, which can be seen from Fig. 6.

Figure 7 shows that the dimensionless concentration profiles for different values of the

Schmidt number Sc . It is clear from this Fig. that the concentration boundary layer thickness increases as the Schmidt number Sc increases and this is the analog to the effect of increasing the Prandtl number on the thickness of a thermal boundary layer.

The effect of chemical reaction parameter K on the concentration profiles is shown in Fig. 8. It can be seen that the concentration profiles increase during the generative reaction ($K < 0$) and decrease in the destructive reaction ($K > 0$). This shows that diffusion rate can be significantly altered by chemical reactions. The effect of thermoporethic parameter τ on the concentration profiles is shown in Fig. 9 from this Fig. it is noticed that the effect of the thermoporethic parameter is to slightly increase slightly the wall slope of the concentration profiles.

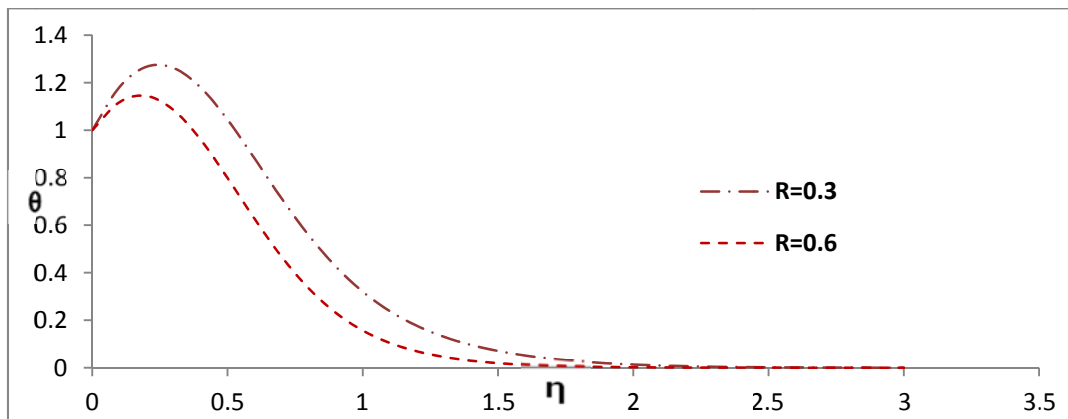


Fig. 5. Temperature profiles for different values of R

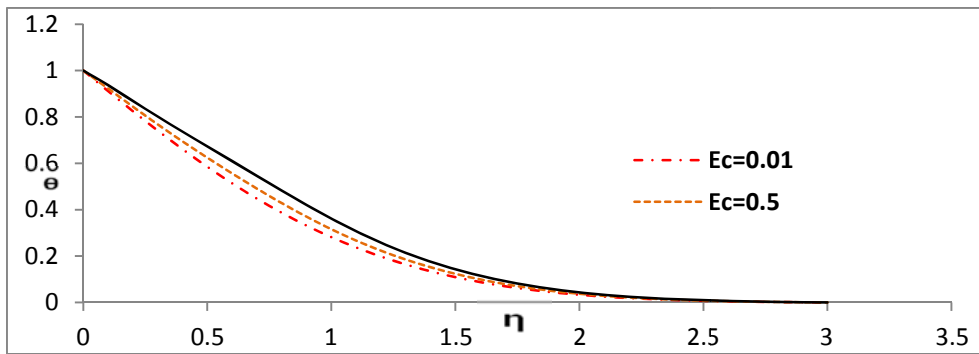


Fig. 6. Temperature profiles for different values of Ec

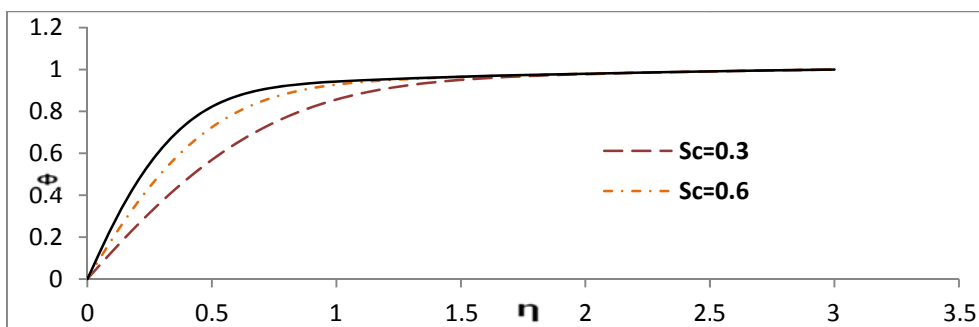


Fig. 7. Temperature profiles for different values of Sc

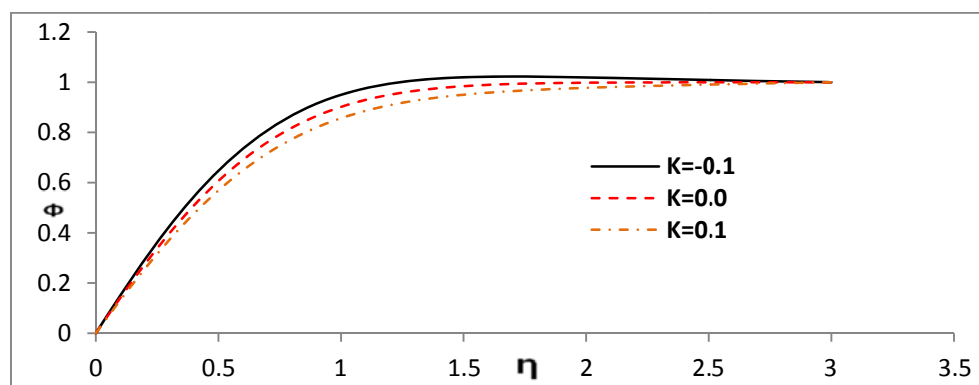


Fig. 8. Concentration profiles for different values of K

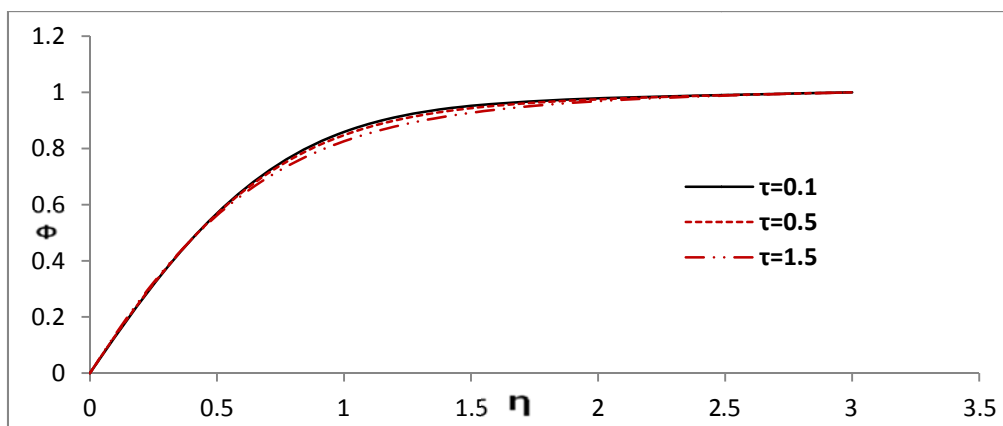


Fig. 9. Concentration profiles for different values of τ

Table 1 illustrates the effects of v_0 and θ_r on the local skin-friction coefficient (C_f), the local Nusselt number (Nu) and the local Stanton number (St). From this Table we see that the local skin-friction coefficient decreases whereas both the local Nusselt number as well as the local Stanton number increases for wall fluid suction ($v_0 > 0$). But imposition of wall fluid injection ($v_0 < 0$) produces the opposite effects compared to the case of wall fluid suction. It is also seen from this Table that both the local skin-friction coefficient (C_f) and the local Nusselt number (Nu) increases with the increasing values of θ_r but the local Stanton number (St) is not sensitive to change as the viscosity/temperature parameter.

Table 1. Effects of v_0 and θ_r on C_f , Nu and St

| v_0 | θ_r | C_f | Nu | St |
|-------|------------|---------|--------|--------|
| -0.5 | 2.0 | -0.4953 | 0.3679 | 0.3843 |
| 0.0 | 2.0 | -1.0061 | 0.4056 | 0.7454 |
| 0.5 | 2.0 | -1.3814 | 0.4463 | 0.9482 |
| 1.5 | 2.0 | -2.1084 | 0.5447 | 1.3551 |
| 1.5 | 4.0 | -1.8741 | 0.5638 | 1.3547 |

Table 2 shows the effects of Pr and R on the local Skin-friction coefficient (C_f), the local Nusselt number (Nu), and the local Stanton number (St). From this Table we see that as the radiation parameter R increases, the local skin-friction coefficient and the local Stanton number decrease while the Nusselt number increases for liquid metals (Pr=0.05), air (Pr=0.70) as well as for water (Pr=7.0)

Table 2. Effects of Pr and R on C_f , Nu and St

| Pr | R | C_f | Nu | St |
|------|-----|---------|---------|--------|
| 0.05 | 0.1 | -0.2909 | 0.0812 | 0.9429 |
| 0.05 | 0.3 | -0.4086 | 0.1138 | 0.9422 |
| 0.70 | 0.1 | -0.7428 | 0.2652 | 0.9840 |
| 0.7 | 0.3 | -0.9371 | 0.4463 | 0.9474 |
| 7.00 | 0.1 | -1.1754 | 0.95034 | 0.9497 |
| 7.00 | 0.3 | -1.3164 | 1.7148 | 0.9488 |

The effects of Schmidt number Sc and thermophoretic parameter τ on the local Stanton number (St) are shown in Table 3. It is observed from this Table that the local Stanton number increases as St or τ increases.

Table 3. Effects of Sc and τ on St

| Sc | T | St |
|------|-----|--------|
| 0.30 | 0.5 | 0.6532 |
| 0.30 | 1.0 | 0.6688 |
| 0.60 | 1.0 | 1.0248 |
| 1.00 | 1.0 | 1.4375 |

Finally in Table 4, the effects of the chemical reaction parameter and the order of the chemical reaction on the local Stanton number (St) are presented. It is observed from this Table that the local Stanton (St) is presented. It is observed from this Table that the local Stanton number decreases with the increase of K for a destructive reaction ($K > 0$) and increases with the decreases of K for a generative reaction ($K < 0$). It is also found from this Table that the local Stanton number increases with the increasing values of n for both destructive as well as generative reaction.

Table 4. Effects of K and n on St

| K | n | St |
|------|-----|--------|
| -0.2 | 1.0 | 1.4186 |
| 0.0 | 1.0 | 1.0823 |
| 0.1 | 1.0 | 0.9484 |
| -0.2 | 3.0 | 1.5135 |
| 0.1 | 3.0 | 0.9948 |

5. Conclusion

A comprehensive numerical parametric study for the numerical solutions of a class of nonlinear equations is conducted and results are reported in terms of graphs. This is done in order to illustrate special features of the solutions. So the numerical solutions by using finite difference method were obtained. Apart from that, obtained results indicate that it is an adequate scheme for the solution of the present problems. The results also show that higher order chemical reaction induces the concentration of the particles for a destructive reaction and reduces for a generative reaction. The results show that variable viscosity significantly increases viscous drag and rate of heat transfer.

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