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Scintillation index effects on the bit error rate in free space optical communication of incoherent flat-topped laser beam propagating through turbulent atmosphere

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Abstract

In this paper, the effects of scintillation phenomena on Bit Error Rate (BER) and Signal to Noise Ratio (SNR), which are essential characteristics in Free Space Optical (FSO) communication system, are studied. Until now various methods have been introduced to decrease BER and increase the link's reliability. Optimizing SNR is one of the fundamental methods to decrease BER value. Intensity fluctuations variance of laser beam on the receiver optics due to propagation through turbulent atmosphere which is called Scintillation Index is an effective parameter in SNR. Thus, due to the importance of this index in FSO systems, to study the effects of source and turbulence parameters on BER, first the scintillation index of incoherent flat-topped laser beam propagating through turbulent atmosphere is calculated (using Kolmogorov spectrum model). Then, an analytical formula for SNR and so BER based on Log-Normal irradiance distribution function is expressed. With the help of this formula, results of investigations are shown by graphs. Simulations show that scintillation index and source parameters such as modulation methods and order of beam's flatness have a strong effect on SNR and BER.

Keywords*:* Bit Error Rate; free space optical communication; log-normal pdf; scintillation index; signal to noise ratio

1. Introduction

Optical communications technology which is based on propagating laser beams in free space, specifically the atmosphere and exchanging data between two points is called Free-Space Optical (FSO) communication. Data transmission with possible minimum Bit Error Rate (BER) is the main purpose of any communication system. Increasing the Signal to Noise Ratio (SNR) is one of the most effective strategies in reducing the BER. In addition to absorption and scattering phenomena, which reduces the intensity of the transmitted beam, random fluctuations of media refractive index (due to variations of pressure, temperature, humidity, wind shear, etc.) also induces atmospheric turbulences which results in intensity fluctuation at the receiver optics. Whereas intensity fluctuations of laser beam after propagation through turbulent atmosphere is due to energy redistribution of beam in its cross-section profile which is called *Scintillation*, and is a key factor in SNR and BER values, a lot of attention has been drawn to the effect of this factor in FSO systems. Theoretical and experimental studies on propagating parameters of laser beam profiles has been reported in much of literatures (Eyyuboglu et al., 2004; Cai et al., 2006). On the other hand, based on recent studies, it has been shown that the influence of atmospheric turbulence on the partially coherent flat-topped beam is less than the full coherent beam (Cai et al., 2006; Wang et al., 1979; Kashani et al., 2009; Alavinejad et al., 2008). The most salient feature of flat-topped laser beams is uniform intensity distribution in the beam cross-section profiles providing much better data modulation compared with conventional Gaussian laser beam. It is therefore important to understand the factors which affect the BER (such as scintillation) and to simulate more precisely the effects of these factors on FSO link's reliability. In this study, the effect of scintillation index on BER based on the incoherent flat-topped laser beam propagating through turbulent atmosphere has been analyzed, using the Kolmogorov spectral model to simulate the effects of atmospheric turbulence. To achieve this goal, after calculation of the scintillation index, an analytical formula for SNR and BER based on Log-Normal intensity distribution function is derived and the effect of changes of this index on BER in free space optical communication link is then investigated. Results of these investigations are shown by graphs.

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2. Scintillation Index and Bit Error Rate of Incoherent Flat-topped Laser Beam

Intensity fluctuations are calculated using normalized intensity variance which is called scintillation index, σ_I^2 (Andrews et al., 2001):

$$\sigma_I^2 = \frac{<I^2>}{<I>^2} - 1 \tag{1}$$

where, *I* is the intensity of the beam at the receiver and < > represents ensemble average. Average value of the SNR according to received intensity and scintillation index is calculated as follows (Andrews et al., 2005):

$$\langle SNR \rangle = \frac{SNR_0}{\sqrt{(\sigma_I^2 SNR_0^2 + I_0 / I)}} \tag{2}$$

where SNR_0 is photon noise and I_0 is the initial laser beam intensity. In order to calculate the BER, after choosing appropriate model for laser beam intensity distribution function in weak atmospheric turbulent condition, the scintillation index of incoherent flat-topped laser beam is obtained first and then, analytical relationship of BER is extracted for several different modulations. The results of the calculations are compared with the case in which Gaussian beam was used.

2.1. Intensity Distribution Model in the Turbulent Atmosphere

Beam intensity fluctuations caused by atmospheric turbulence are modeled as probability density function of the intensity and the BER is calculated based on this probability function. Various models have been introduced to describe the distribution of intensity fluctuations on receiver plane for laser beam propagating through turbulent atmosphere. The most important ones are: normal logarithmic (Log-Normal), Gamma - Gamma and negative exponential models (Negative Exponential) (Andrews, 2004). The normal logarithmic model used in this paper has a relatively high accuracy in the simulation of the intensity distribution in weak atmospheric turbulence. Intensity probability density function of this model is described as follows (Andrews, 2004):

$$p(I) = \frac{1}{I\sqrt{2\pi\sigma_I^2}} exp\left\{ -\frac{\left[\ln\left(\frac{I}{I_0}\right) + \left(\frac{1}{2}\right)\sigma_I^2\right]^2}{2\sigma_I^2} \right\}, I \geq 0 \tag{3}$$

In Fig. 1, the probability density function of this model is shown for different scintillation indices.

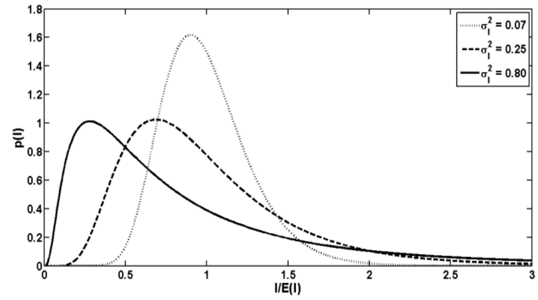

Fig. 1. The probability density function of the normalized intensity in the normal logarithmic model for different scintillation indices

As seen in Fig. 1, by increasing the scintillation index, the maximum value of probability density function occurs at lower normalized intensities in small values and the probability density function is broadened.

2.2. Incoherent Flat-topped Laser Beam

In the direct detection optical receivers, the beam intensity plays a significant role. Thus decreasing the radiation fluctuations and scintillation index will play an important role in improving the BER. Using the appropriate laser beam profile is one of the important methods for improving propagation properties of laser beam and link parameters. Because of uniform cross section profile, the flat-topped laser beam is less affected by turbulent atmosphere in comparison with Gaussian beam. The electric field of a flat-topped laser beam with circular symmetry in the plane source, z=0, can be expressed as a finite sum of elliptical Gaussian beams in the orthogonal coordinate system (Li, 2002):

$$u_d(\vec{s}) = \sum_{n=1}^{N} \frac{(-1)^{n-1}}{N} \binom{N}{n} \exp\left(-\frac{n|\vec{s}|^2}{2\alpha_s^2}\right) \tag{4}$$

where $\vec{s} = (s_x, s_y)$ and α_s^2 are source transverse coordinates and elliptical Gaussian beam size, respectively and *N* is the number of Gaussian beam sources which are assembled together to produce uniform profile and is called order of flatness. The full coherent laser beam propagating through turbulent media can be experienced through some light fringes patterns (Francon, 1979), which can lead to error in receiving data. Therefore, for data transmission, incoherent flat-topped laser beam can be used. Incoherent effect can be expressed mathematically by multiplying the beam optical field in a random factor.

$$u(\vec{s}) = u_d(\vec{s})u_r(\vec{s}) \tag{5}$$

where $u_r(\vec{s}) = \exp\left[i\theta(s)\right]$ and θ are randomized phase of source function. Based on the extended Huygens-Fresnel principle, optical field in receiver plane is described as follows (Fante, 1985):

$$u(\vec{p}) = \frac{\exp(ikL)}{\lambda iL}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} d^2s\, u(\vec{s}) \times\exp\left[\frac{ik}{2L}|\vec{p}-\vec{s}|^2 + \psi(\vec{s},\vec{p})\right] \tag{6}$$

where L is link path length and $\psi(\vec{s},\vec{p})$ is the random part of the complex phase of a spherical wave propagating in the turbulent medium from the point $(\vec{s},0)$ to the point $(\vec{p},L)$. Using a receiver whose response time is much longer than the laser beam temporal coherence, the beam intensity can be defined analytically as follows (Eyyuboglu et al., 2004):

$$\begin{aligned} I(\vec{p},L) &= u(\vec{p})u^*(\vec{p}) \\ &= \frac{1}{(\lambda L)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2s_1 d^2s_2 \\ &\times\Gamma_2^s(\vec{s}_1,\vec{s}_2) \\ &\times\exp\left\{\frac{ik}{2L}\left[|\vec{p}-\vec{s}_1|^2 - |\vec{p}-\vec{s}_2|^2\right]\right\} \\ &\times\exp\left[\psi(\vec{s}_1,\vec{p}) + \psi^*(\vec{s}_2,\vec{p})\right] \end{aligned} \tag{7}$$

where Γ_2^s is cross-correlation function and can be expressed as follows (Goldsmith, 2008):

$$\Gamma_2^s(\vec{s}_1,\vec{s}_2) = \lambda^2 I\left[\frac{\vec{s}_1+\vec{s}_2}{2}\right]\delta(\vec{s}_1-\vec{s}_2) \tag{8}$$

In equation (8), δ is the Dirac function. Based on optical field second order statistical moment, the average beam intensity on the receiver is calculated using equations (4) to (8) and is presented as:

$$\begin{aligned} \langle I(\vec{p},L)\rangle &= \langle u_1(\vec{p})u_2^*(\vec{p})\rangle \\ &= \frac{2\pi\alpha_s^2}{(LN)^2}\sum_{n=1}^{N}\sum_{n'=1}^{N}\frac{(-1)^{n+n'}}{(n+n')}\binom{N}{n}\binom{N}{n'} \end{aligned} \tag{9}$$

According to equation (1), to obtain the scintillation index the average squared intensity should be calculated. For this purpose, using field fourth order statistical moment, $\langle I^2(\vec{p},L)\rangle$ is obtained as follows:

$$\begin{aligned} &\langle I^2(\vec{p},L)\rangle = \langle u_1(\vec{p})u_2^*(\vec{p})u_3(\vec{p})u_4^*(\vec{p})\rangle \\ &= \frac{1}{(\lambda L)^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2s_1 d^2s_2 d^2s_3 d^2s_4 \\ &\times\Gamma_2^s(\vec{s}_1,\vec{s}_2)\Gamma_2^s(\vec{s}_3,\vec{s}_4) \\ &\times\exp\left\{\frac{ik}{2L}\left[|\vec{p}-\vec{s}_1|^2 - |\vec{p}-\vec{s}_2|^2 + |\vec{p}-\vec{s}_3|^2 - |\vec{p}-\vec{s}_4|^2\right]\right\} \\ &\times\exp\left[\psi(\vec{s}_1,\vec{p}) + \psi^*(\vec{s}_2,\vec{p}) + \psi(\vec{s}_3,\vec{p})] + \psi^*(\vec{s}_4,\vec{p})\right] \end{aligned} \tag{10}$$

With the change of variables of $\psi(\vec{s},\vec{p}) = \chi(\vec{s},\vec{p}) + iS(\vec{s},\vec{p})$, where χ is called the log amplitude and S is the phase, and after tedious but straightforward computation, finally the mean square of intensity, $\langle I^2(\vec{p},L)\rangle$, calculated as follows:

$$\begin{aligned} &\langle I^2(\vec{p},L)\rangle \cong \langle I(\vec{p},L)\rangle^2 + \frac{4\pi^2\sigma_x^2}{(LN)^4} \\ &\times\sum_{n=1}^{N}\sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{m=1}^{N}\frac{(-1)^{n+m+n'+m'}\binom{N}{n'}\binom{N}{n}\binom{N}{m}\binom{N}{m'}4\alpha_s^2}{(n+n')(m+m') + \frac{2\alpha_s^2}{\rho_0^2}(m+m'+n+n')} \end{aligned} \tag{11}$$

In the equation (11), in order to simulate the effects of atmospheric turbulence on laser beam propagation, Kolmogorov spectral model is used, accordingly, $\rho_0 = \left(0.545C_n^2k^2L\right)^{-3/5}$ is the effective length of coherence in the atmosphere, k is propagation constant and C_n^2 is atmospheric refractive-index structure parameter and $\sigma_x^2 = 0.124C_n^2k^{7/6}L^{11/6}$ is variance of log-amplitude fluctuations for the spherical wave. Scintillation index will be calculated using equations (1), (9) and (11). In Fig. 2, scintillation index is simulated for different laser beam width for incoherent beam. As can be seen, where the order of flatness of the laser beam is higher, the scintillation index is lower and by increasing the width of the beam, scintillation index decreases. It is worth mentioning that increasing order of flatness leads to an increase in intensity uniformity and beam width value.

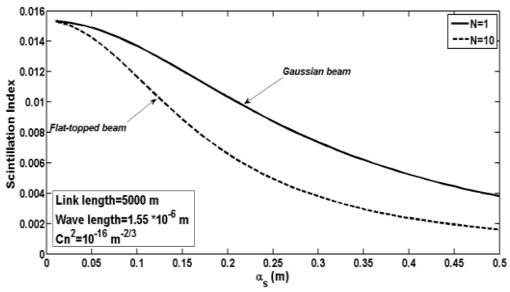

Fig. 2. Scintillation index for different beam width of incoherent Gaussian (with flatness degree of 1) and flat-topped (with flatness degree of 10) laser beams

In Fig. 3, scintillation index of incoherent Gaussian and flat-topped laser beams based on link path length is shown. By increasing the length of link, the scintillation index of incoherent Gaussian beam is increased considerably compared with the flat-topped laser beam and therefore, it will have greater error.

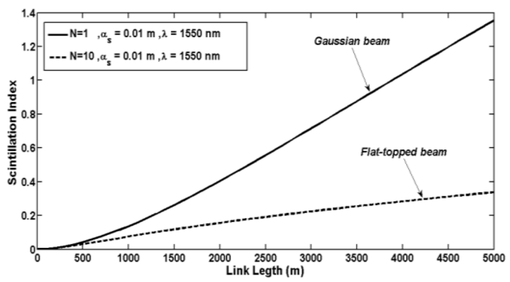

Fig. 3. Scintillation index as function of the length of link

According to Figs. 2 and 3, it can be deduced that incoherent flat-topped beam scintillates less than the Gaussian beam through atmospheric turbulence and hence, its utilization in free space optical communication data transmission is more suitable.

2.3. Bit Error Rate

To obtain the BER, the average value of the SNR should be calculated from equation (2). For the On-Off Keying (OOK) modulation, the error rate is given by (Ghassemlooy et al., 2012):

$$P_e = Q\left(\frac{SNR}{\sqrt{2}}\right) \quad (12)$$

and also, in Binary Phase Shift Keying (BPSK) modulation, the error rate is given by (Eyyuboglu et al., 2004):

$$P_e = Q\left(\sqrt{2}SNR\right) \quad (13)$$

The error rate by OOK modulation in atmospheric turbulence Log-Normal model is calculated as follows:

$$\Pr(E) = BER = \frac{1}{2}\int_0^{\infty} p_I(u)\ erfc\left(\frac{SNRu}{2\sqrt{2}}\right)du \quad (14)$$

where:

$$erfc\left(\frac{SNR}{2}\right) = 2Q(SNR/\sqrt{2}) \quad (15)$$

The BER is calculated by replacing the probability density function (equation (3)) in equation (14). To achieve this goal, the use of Hermit-Gaussian approximation is essential. The Hermit–Gaussian approximation is as follows:

$$\int_{-\infty}^{\infty} f(x)exp(-x^2)dx \cong \sum_{i=1}^{n} w_i f(x_i) \quad (16)$$

where:

$$w_i = \frac{2^{n-1} n!\sqrt{\pi}}{n^2\left[H_{n-1}(x_i)\right]^2} \quad (17)$$

and x_i represents the Hermit polynomial roots. Also, the Q function should be introduced as follows:

$$Q(x) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{x^2}{2sin^2(\theta)}\right)d\theta \ \text{ for } x > 0 \quad (18)$$

where $x = \dfrac{\ln(u) + \left(\frac{1}{2}\right)\sigma_l^2}{\sqrt{2\sigma_l^2}}$.

Finally, the BER is given as:

$$BER \cong \frac{1}{\sqrt{\pi}}\sum_{i=1}^{n} w_i \mathrm{Q}\left(\frac{SNR}{\sqrt{2}}\exp\left(\sqrt{2\sigma_l^2}\ x_i - \frac{\sigma_l^2}{2}\right)\right) \quad (19)$$

In Fig. 4 the BER is simulated for Gaussian and flat-topped beams with different degrees of flatness and for two types of OOK and BPSK modulation.

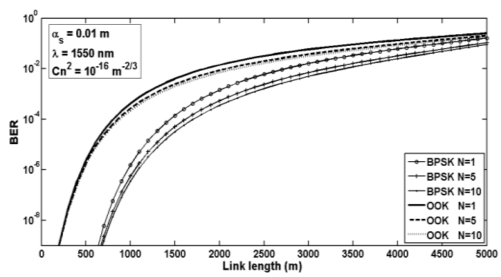

Fig. 4. Bit error rate of Log-normal turbulence model for Gaussian and flat-topped laser beams with order of flatness of 5 and 10 in OOK and BPSK modulation models

According to this figure, BER for flat-topped incoherent beams with higher flat topped order is less than lower order of flatness. However, due to the use of incoherent beams for data transmission, its broadening is more than coherent beam and thus the less intensity is collected on the detector. Therefore, laser beam will experience a smaller scintillation index during propagation in the turbulent atmosphere. In Fig. 5, the effect of scintillation index changes on the BER in free space optical communication systems has been studied. As it is shown, the reduction in scintillation index causes an increase in SNR value and thereby a decrease in BER at data transmission in optical communication systems.

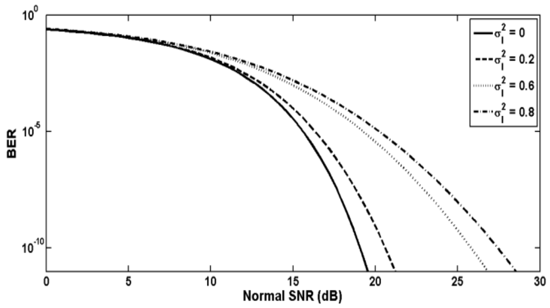

Fig. 5. Flat-topped beam BER with flatness degree of 10 based on SNR for different scintillation indices

2.4. Signal constellation

In multi-level modulation, signal constellation provides a better understanding of the BER situation. In Fig. 6 signal constellation is compared for two states of signal with order of flatness of 10 and 1. In the case that laser beam has less order of flatness in turbulent atmosphere, distribution of the received bits in four different modes of bit have more overlap, and thus will increase the likelihood of error.

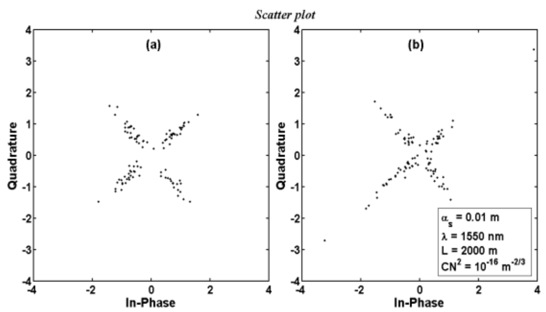

Fig. 6. Scatter plot of signal system for BPSK modulation. (a) *N=10*, (b) *N=1*

3. Conclusion

In this paper by calculating the scintillation index of incoherent flat topped laser beam propagating through turbulent atmosphere, an analytical relationship of bit error rate were extracted for normal logarithm intensity distribution models for different modulations. Then, effects of changes of scintillation index on the bit error rate were examined. The results are compared with the case that Gaussian laser beam was used in optical communication system and was represented with the help of figures obtained from simulation calculations. It is found that:

1. Reducing the scintillation index will increase the SNR and will thus reduce the value of BER.
2. By using incoherent flat-topped laser beam, the scintillation index can be reduced. As scintillation index and intensity are two main parameters in BER value, any decrease in scintillation index value causes a decrease in BER amount. However, due to the fact that broadening of incoherent flat topped beam is more than the coherent, the beam intensity is less than Gaussian beam.
3. The higher the order of flatness, the lower the bit error rates for incoherent flat-topped laser beam.

Analytical results obtained can be used in many laser applications based on flat-topped laser beam propagation in the atmosphere, especially in free space optical communication systems.

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