IJST (2015) 39A3: 281-288
Iranian Journal of Science & Technology

http://ijsts.shirazu.ac.ir

Numerical solution of the Rosenau equation using quintic collocation B-spline method

Rasoul Abazari[1]* and Reza Abazari[2]

[1]*Department of Mathematics, Ardabil Branch, Islamic Azad University, Ardabil, Iran*
[2]*Young Researchers and Elite Club, Ardabil Branch, Islamic Azad University, Ardabil, Iran*
[2]*Department of Mathematics, Behbahan Khatam Alanbia University of Technology, Behbahan, Iran*
E-mails: rasoolabazari@gmail.com & abazari.r@gmail.com

Abstract

In this paper, the quintic B-spline collocation scheme is employed to approximate numerical solution of the KdV-like Rosenau equation. This scheme is based on the Crank-Nicolson formulation for time integration and quintic B-spline functions for space integration. The unconditional stability of the present method is proved using Von-Neumann approach. Since we do not know the exact solution of the nonlinear KdV-like Rosenau equation, a comparison between the numerical solutions on a coarse mesh and those on a refine mesh is made to show the efficiency of discussed method.

***Keywords**:* Rosenau equation; Quintic B-spline method; Crank-Nicolson scheme; Thomas algorithm

1. Introduction

In 1895, Dutch physicist Diederick Korteweg and his student Gustav de Vries (Korteweg, D. J. et al 1895), derived the famous equation, namely KdV equation, to study the propagation of waves in one dimension on the surface of water. The KdV equation is a balance between time evolution, nonlinearity and dispersion of waves in one dimension on the surface of water. This equation is one of the famous nonlinear equations for solitary waves, and is one of the simplest and most useful nonlinear model equations to study the dynamics of dense discrete systems (Rosenau, 1986, Rosenau, 1988). In the study of the dynamics of dense discrete systems, specially the cases of wave-wave and wave-wall, interactions cannot be described using the well-known KDV equation. To overcome this shortcoming of the KDV equation, Rosenau (Rosenau, 1986, Rosenau, 1988) proposed the so-called Rosenau equation:

$$u_t + u_{xxxxt} + u_x + uu_x = 0, \quad x \in \Omega, \, t \in (0,T], \qquad (1)$$

with the boundary conditions

$$u(x,t) = u_{xx}(x,t) = 0, \quad x \in \partial\Omega, \, t \in (0,T], \qquad (2)$$

and an initial condition

$$u(x,0) = u_0(x), \quad x \in \overline{\Omega}, \qquad (3)$$

where $u_0(x)$ is sufficiently smooth and satisfies the compatibility condition, $\Omega = (0,1)$ and $0 < T < +\infty$. For further physical significance of the Rosenau equation (1), we refer to papers (Rosenau, 1986) and the references given therein.

The global existence and the uniqueness of the solution for Eq. (1) was proved by Park (Park, 1990). But it is difficult to find the analytical solution for Eq. (1). Since then, much work has been done by using some different numerical methods to approximate solution of Eq. (1) (Chung, 1998, Chung, et al. 2001, Manickam, et al. 1998, Kim, et al. 1998) and also the references therein.

In this paper, the quintic B-spline collocation scheme is employed to approximate numerical solution of the KdV-like Rosenau equation (1). This scheme is based on the Crank-Nicolson formulation for time integration and quintic B-spline functions for space integration. The present scheme will be used first to construct a numerical model for the KdV-like Rosenau equation (1) and then its results will implement to approximate the numerical solution of (1).

The quintic B-spline basis has been used to build up the approximation solutions for some nonlinear differential equations. For instance, numerical solution of the Burger equation has been found by quintic B-spline collocation method in (Sepehrian, et al. 2008). An algorithm based on quintic B-spline Galerkin method was set up to obtain the solutions of the RLW equation in

*Corresponding author
Received: 22 April 2014 / Accepted: 17 June 2015

(Dag, et al. 2006). Collocation of quintic B-spline interpolation functions over finite elements was described to approximate the numerical solution of the Korteweg-de Vries (KdV) equation and KdV-Burgers equation in (Zaki et al. 2000, Gardner et al. 1990, Lee, et al. 1997), respectively.

The organization of this paper is as follows. In Section 2, quintic B-spline collocation scheme is explained. In Sections 3 the method is applied to the KdV-like Rosenau equation (1). In Section 4, the stability analysis of the method is discussed. In Section 5, one examples is presented. Also the global relative error at different time is obtained for the example. An overall summary of the present work is given at the end of the paper in Section 6.

2. Description of the quintic B-spline method

The solution domain $0 \le x \le 1$ is partitioned in to a mesh of uniform length $h = x_{i+1} - x_i$, by knots x_i where $i = 0,1,2,\cdots,N$ such that $0 = x_0 < x_1 < \cdots < x_{n-1} < x_N = 1.$ Our numerical treatment for Rosenau equation using the collocation method with quintic B-spline is to find an approximate solution $U_N(x,t)$ to the exact solution $u(x,t)$ in the form:

$$U_N(x,t) = \sum_{i=-2}^{N+2} \delta_i(t) B_i(x), \tag{4}$$

where $\delta_i(t)$ are time-dependent quantities to be determined from the boundary conditions and collocation form of the differential equations, and $B_i(x)$ are the quintic B-spline basis functions at knots, given by

$$B_i(x) = \frac{1}{h^5}\begin{cases} (x - x_{i-3})^5, & x \in [x_{i-3}, x_{i-2}) \\ (x - x_{i-3})^5 - 6(x - x_{i-2})^5, & x \in [x_{i-2}, x_{i-1}) \\ (x - x_{i-3})^5 - 6(x - x_{i-2})^5 + 15(x - x_{i-1})^5, & x \in [x_{i-1}, x_i) \\ (x_{i+3} - x)^5 - 6(x_{i+2} - x)^5 + 15(x_{i-1} - x)^5, & x \in [x_i, x_{i+1}) \\ (x_{i+3} - x)^5 - 6(x_{i+2} - x)^5, & x \in [x_{i+1}, x_{i+2}) \\ (x_{i+3} - x)^5, & x \in [x_{i+2}, x_{i+3}) \\ 0, & \text{otherwise} \end{cases}, \tag{5}$$

where $\{B_{-2}, B_{-1}, B_0, B_1, B_2, \ldots, B_{N+1}, B_{N+2}\}$ forms a basis over the region $0 \le x \le 1$. Each quintic B-spline covers six elements so that an element is covered by six quintic B-splines. Over the element $[x_m, x_{m+1}]$ the variation of the function $U(x,t)$ is formed from

$$U(x,t) = \sum_{j=m-2}^{m+3} \delta_j(t) B_j(x), \tag{6}$$

In terms of a local coordinate system ξ given by $h\xi = x - x_m$, where $h = x_{m+1} - x_m$ and $0 \le \xi \le 1$, expressions for the element splines are [10]

$$\begin{aligned} B_{m-2}(x) &= 1 - 5\xi + 10\xi^2 - 10\xi^3 + 5\xi^4 - \xi^5, \\ B_{m-1}(x) &= 26 - 50\xi + 20\xi^2 + 20\xi^3 - 20\xi^4 + 5\xi^5, \\ B_m(x) &= 66 - 60\xi^2 + 30\xi^4 - 10\xi^5, \\ B_{m+1}(x) &= 26 + 50\xi + 20\xi^2 - 20\xi^3 - 20\xi^4 + 10\xi^5, \\ B_{m+2}(x) &= 1 + 5\xi + 10\xi^2 + 10\xi^3 + 5\xi^4 - 5\xi^5, \\ B_{m+3}(x) &= \xi^5. \end{aligned} \tag{7}$$

Using approximate function (4) and quintic spline (5), the approximate values at the knots of $U(x)$ and its derivatives up to fourth order are determined in terms of the time parameters δ_m as

$$\begin{aligned} U_m &= \delta_{m+2} + 26\delta_{m+1} + 66\delta_m + 26\delta_{m-1} + \delta_{m-2}, \\ hU'_m &= 5(\delta_{m+2} + 10\delta_{m+1} - 10\delta_{m-1} - \delta_{m-2}), \\ h^2U''_m &= 20(\delta_{m+2} + 2\delta_{m+1} - 6\delta_m + 2\delta_{m-1} + \delta_{m-2}), \\ h^3U'''_m &= 60(\delta_{m+2} - 2\delta_{m+1} + 2\delta_{m-1} - \delta_{m-2}), \\ h^4U^{(iv)}_m &= 120(\delta_{m+2} - 4\delta_{m+1} + 6\delta_m - 4\delta_{m-1} + \delta_{m-2}), \end{aligned} \tag{8}$$

where dashes represent differentiation with respect to space variable.

3. Solution of Rosenau equation

The Rosenau equation can be rewritten as

$$(u+u_{xxxx})_t + uu_x + u_x = 0, \tag{9}$$

with the boundary conditions

$$\begin{aligned} u(0,t) &= u(1,t) = 0, \\ u_{xx}(0,t) &= u_{xx}(1,t) = 0, \end{aligned} \tag{10}$$

and initial condition

$$u(x,0) = u_0(x), \tag{11}$$

where $(x,t) \in [0,1] \times (0,T]$. We discrete the time derivative of Eq. (9) by a first order accurate forward difference formula and apply the θ-weighted scheme, $(0 \le \theta \le 1)$, to the space derivative at two adjacent time levels to obtain the equation

$$\begin{aligned} &\frac{\left(U^{n+1} + (U_{xxxx})^{n+1}\right) - \left(U^n + (U_{xxxx})^n\right)}{k} \\ &+\theta\left\{(UU_x)^{n+1} + (U_x)^{n+1}\right\} \\ &+(1-\theta)\left\{(UU_x)^n + (U_x)^n\right\} = 0, \end{aligned} \tag{12}$$

where k is time step and the superscripts n and $n+1$ are successive time levels. In this work we take $\theta = \frac{1}{2}$, . Hence, Eq. (12) takes the form

$$\begin{aligned} &\frac{\left(U^{n+1} + (U_{xxxx})^{n+1}\right) - \left(U^n + (U_{xxxx})^n\right)}{k} \\ &+\frac{(UU_x)^{n+1} + (UU_x)^n}{2} + \frac{(U_x)^{n+1} + (U_x)^n}{2} = 0, \end{aligned} \tag{13}$$

The nonlinear term in Eq. (13) is approximated by the following formula based on Taylor series:

$$(UU_x)^{n+1} = U^{n+1}(U_x)^n + U^n(U_x)^{n+1} - (UU_x)^n, \tag{14}$$

Putting values from Eq. (14) in Eq. (13) we get,

$$\begin{aligned} &\frac{\left(U^{n+1} + (U_{xxxx})^{n+1}\right) - \left(U^n + (U_{xxxx})^n\right)}{k} \\ &+\frac{U^{n+1}(U_x)^n + U^n(U_x)^{n+1}}{2} \\ &+\frac{(U_x)^{n+1} + (U_x)^n}{2} = 0, \end{aligned} \tag{15}$$

Rearranging the terms and simplifying we get,

$$\begin{aligned} &U^{n+1} + (U_{xxxx})^{n+1} \\ &+\frac{k}{2}\left\{U^{n+1}(U_x)^n + U^n(U_x)^{n+1} + (U_x)^{n+1}\right\} \\ &= U^n + (U_{xxxx})^n - \frac{k}{2}(U_x)^n, \end{aligned} \tag{16}$$

Substituting the approximate solution U for u and putting the values of the nodal values U, its derivatives using Eqs. (8) at the knots in Eq. (16) yields the following difference equation with the variables δ_i and for $m = 0,1,2,\ldots,N$:

$$\begin{aligned} &C_2\delta_{m+2}^{n+1} + C_1\delta_{m+1}^{n+1} + C_0\delta_m^{n+1} + C_{-1}\delta_{m-1}^{n+1} + C_{-2}\delta_{m-2}^{n+1} \\ &= \overline{C}_2\delta_{m+2}^n + \overline{C}_1\delta_{m+1}^n + \overline{C}_0\delta_m^n + \overline{C}_{-1}\delta_{m-1}^n + \overline{C}_{-2}\delta_{m-2}^n, \end{aligned} \tag{17}$$

where

$$\begin{aligned} C_2 &= 1 + \frac{120}{h^4} + \frac{k}{2}U_x^n + \frac{k}{2}\frac{5}{h}(U^n + 1), \\ C_1 &= 26 - 4\frac{120}{h^4} + \frac{k}{2}(26U_x^n) + \frac{k}{2}\frac{5}{h}(10U^n + 10), \\ C_0 &= 66 + 6\frac{120}{h^4} + \frac{k}{2}(66U_x^n), \\ C_{-1} &= 26 - 4\frac{120}{h^4} + \frac{k}{2}(26U_x^n) - \frac{k}{2}\frac{5}{h}(10U^n + 10), \\ C_{-2} &= 1 + \frac{120}{h^4} + \frac{k}{2}U_x^n - \frac{k}{2}\frac{5}{h}(U^n + 1), \end{aligned} \tag{18}$$

and

$$
\begin{aligned}
\overline{C}_2 &= 1+\frac{120}{h^4}-\frac{k}{2}\frac{5}{h},\\
\overline{C}_1 &= 26-4\frac{120}{h^4}-10\frac{k}{2}\frac{5}{h},\\
\overline{C}_0 &= 66+6\frac{120}{h^4},\\
\overline{C}_{-1} &= 26-4\frac{120}{h^4}+10\frac{k}{2}\frac{5}{h},\\
\overline{C}_{-2} &= 1+\frac{120}{h^4}+\frac{k}{2}\frac{5}{h},
\end{aligned} \tag{19}
$$

The system (17) consists of $(N+1)$ linear equations in $(N+5)$ unknowns $(\delta_{-2},\delta_{-1},\delta_0,\delta_1,\delta_2,...,\delta_{N-1},\delta_N,\delta_{N+1},\delta_{N+2})^T$, To obtain a unique solution to the system (17), four additional constraints are required. These are obtained from the boundary conditions (10). Imposition of the boundary conditions enables us to eliminate the parameters δ_{-2},δ_{-1} and $\delta_{N+1},\delta_{N+2}$ from the system. In order to eliminate the parameters δ_{-2},δ_{-1} and $\delta_{N+1},\delta_{N+2}$ from the system (17), we have used the boundary conditions

$$
\begin{aligned}
u(x_0,t) &= u(x_N,t)=0,\\
u_{xx}(x_0,t) &= u_{xx}(x_N,t)=0,
\end{aligned}
$$

Expanding u in terms of approximate quintic B-spline formula from (8) at $x_0=0,$ and putting $m=0$ in (8) we get,

$$
\begin{aligned}
&\delta_2+26\delta_1+66\delta_0+26\delta_{-1}+\delta_{-2}=0,\\
&\delta_2+2\delta_1-6\delta_0+2\delta_{-1}+\delta_{-2}=0,
\end{aligned} \tag{20}
$$

then

$$
\begin{aligned}
\delta_{-1} &= -3\delta_0-\delta_1,\\
\delta_{-2} &= 12\delta_0-\delta_2,
\end{aligned} \tag{21}
$$

Similarly at $x_N=1,$ putting $m=N$ in (8) we get,

$$
\begin{aligned}
&\delta_{N+2}+26\delta_{N+1}+66\delta_N+26\delta_{N-1}+\delta_{N-2}=0,\\
&\delta_{N+2}+2\delta_{N+1}-6\delta_N+2\delta_{N-1}+\delta_{N-2}=0,
\end{aligned} \tag{22}
$$

which leads to

$$
\begin{aligned}
\delta_{N+1} &= -3\delta_N-\delta_{N-1},\\
\delta_{N+2} &= 12\delta_N-\delta_{N-2},
\end{aligned} \tag{23}
$$

Eliminating parameters δ_{-2},δ_{-1} and $\delta_{N+1},\delta_{N+2}$ the system (17) is reduced to a penta-diagonal system of $(N+1)$ linear equations with $(N+1)$ unknowns, given by $AX_{n+1}=\overline{A}X_n$ where

$$
\begin{aligned}
X_{n+1} &= (\delta_0^{n+1},\delta_1^{n+1},\delta_2^{n+1},...,\delta_{N-1}^{n+1},\delta_N^{n+1})^T,\\
X_n &= (\delta_0^{n},\delta_1^{n},\delta_2^{n},...,\delta_{N-1}^{n},\delta_N^{n})^T,
\end{aligned}
$$

and T stands for transpose. The coefficient matrix A is given by

$$
A=\begin{pmatrix}
12C_{-2}-3C_{-1}+C_0 & C_1-C_0 & C_2-C_{-2} & 0 & 0 & 0 & 0\\
C_0-3C_{-2} & C_0-C_{-2} & C_1 & C_2 & 0 & 0 & 0\\
C_{-2} & C_{-1}0 & C_0 & C_1 & C_2 & 0 & 0\\
0 & C_{-2} & C_{-1} & C_0 & C_1 & C_2 & 0\\
0 & 0 & \ddots & \ddots & \ddots & \ddots & 0\\
0 & 0 & C_{-2} & C_{-1} & C_0 & C_1 & C_2\\
0 & 0 & 0 & C_{-2} & C_{-1} & C_0-C_2 & C_1-3C_2\\
0 & 0 & 0 & 0 & C_{-2}-C_2 & C_0-C_1 & C_0-3C_1+12C_2
\end{pmatrix} \tag{24}
$$

where C_{-2},C_{-1},C_0,C_1 and C_2 are given in (18), and the coefficient matrix $\overline{A}$, is

$$\overline{A}=\begin{pmatrix} 12\overline{C}_{-2}-3\overline{C}_{-1}+\overline{C}_{0} & \overline{C}_{1}-\overline{C}_{0} & \overline{C}_{2}-\overline{C}_{-2} & 0 & 0 & 0 & 0\\ \overline{C}_{0}-3\overline{C}_{-2} & \overline{C}_{0}-\overline{C}_{-2} & \overline{C}_{1} & \overline{C}_{2} & 0 & 0 & 0\\ \overline{C}_{-2} & \overline{C}_{-1} & \overline{C}_{0} & \overline{C}_{1} & \overline{C}_{2} & 0 & 0\\ 0 & \overline{C}_{-2} & \overline{C}_{-1} & \overline{C}_{0} & \overline{C}_{1} & \overline{C}_{2} & 0\\ 0 & 0 & \ddots & \ddots & \ddots & \ddots & 0\\ 0 & 0 & \overline{C}_{-2} & \overline{C}_{-1} & \overline{C}_{0} & \overline{C}_{1} & \overline{C}_{2}\\ 0 & 0 & 0 & \overline{C}_{-2} & \overline{C}_{-1} & \overline{C}_{0}-\overline{C}_{2} & \overline{C}_{1}-3\overline{C}_{2}\\ 0 & 0 & 0 & 0 & \overline{C}_{-2}-\overline{C}_{2} & \overline{C}_{0}-\overline{C}_{1} & \overline{C}_{0}-3\overline{C}_{1}+12\overline{C}_{2} \end{pmatrix} \quad (25)$$

where $\overline{C}_{-2},\overline{C}_{-1},\overline{C}_{0},\overline{C}_{1}$ and $\overline{C}_{2}$ are also given in (18). This penta-diagonal system can be solved by a modified form of Thomas algorithm. The time evolution of the approximate solution $U_N(x,t)$ is determined by the time evolution of the vector X_N^n which is found repeatedly by solving the recurrence relation, once the initial vectors X_N^0 have been computed from the initial and boundary conditions.

3.1. The initial state

The initial vector X_N^0 can be determined from the initial condition $u(x,0)=u_0(x)$ which gives $(N+1)$ equation in $(N+5)$ unknowns. For the determination of the unknowns relations at the knot the boundary conditions are used (10). The initial vector is then determined as the solution of the matrix equation $A_N^0 X_N^0 = u_0(x)$, where

$$A_N^0=\begin{pmatrix} 54 & 60 & 6 & 0 & 0 & 0 & 0\\ \frac{101}{4} & \frac{135}{2} & \frac{105}{4} & 1 & 0 & 0 & 0\\ 1 & 26 & 66 & 26 & 1 & 0 & 0\\ 0 & 1 & 26 & 66 & 26 & 1 & 0\\ 0 & 0 & \ddots & \ddots & \ddots & \ddots & 0\\ 0 & 0 & 1 & 26 & 66 & 26 & 1\\ 0 & 0 & 0 & 1 & \frac{105}{4} & \frac{135}{2} & \frac{101}{4}\\ 0 & 0 & 0 & 0 & 6 & 60 & 54 \end{pmatrix},$$

and

$X_N^0=(\delta_0^0,\delta_1^0,\delta_2^0,\cdots,\delta_{N-1}^0,\delta_N^0)^T$,

$u_0(x)=(u_0(x_0),u_0(x_1),\cdots,u_0(x_{N-1}),u_0(x_N))^T$,

where $u_0(x_i), i=0,1,2,....N$ can be obtained by initial condition (11).

4. Stability of the proposed scheme

Von-Neumann stability method is used for the stability of scheme developed in the previous section. To apply this method, we have linearized the non-linear term UU_x by considering U as a constant in (14), therefore $U_x, U_{xx},...=0.$ Now substitute, $\delta_m^n=\xi^n\exp(i\rho mh)$ into linearized form of (14), where ρ and h are the mode number and element size, respectively, and $i=\sqrt{-1}$, Eq. (17) leads to

$$\xi\{C_2e^{2i\rho h}+C_1e^{i\rho h}+C_0+C_{-1}e^{-i\rho h}+C_{-2}e^{-2i\rho h}\} \\ =\overline{C}_2e^{2i\rho h}+\overline{C}_1e^{i\rho h}+\overline{C}_0+\overline{C}_{-1}e^{-i\rho h}+\overline{C}_{-2}e^{-2i\rho h}, \quad (26)$$

Here C_j and $\overline{C}_j$, for $j=-2,-1,0,1,2$ have their predefined definition given in (18). Set $X=\frac{120}{h^4}$, $Y=\frac{k}{2}U_x^n$, $Z=\frac{k}{2}\frac{5}{h}U^n$ and $W=\frac{k}{2}\frac{5}{h}$. Simplifying Eq. (26), we get

$$\xi=\frac{a-ib_1}{a+ib_2}, \quad (27)$$

where

$$a=(2+2X)\cos(2\rho h)+(52-8X)\cos(\rho h)$$
$$+66+6X,$$
$$b_1=2W\sin(2\rho h)+20W\sin(\rho h), \quad (28)$$
$$b_2=2(Z+W)\sin(2\rho h)+20(Z+W)\sin(\rho h),$$

From (28), we get
$b_2=b_1+2Z\sin(2\rho h)+20Z\sin(\rho h),$
therefore $a^2+b_1^2\le a^2+b_2^2$. This implies $\|\xi\|\le 1$, which is the condition for scheme to be unconditionally stable.

5. Numerical computations

Consider the Rosenau equation

$$(u+u_{xxxx})_t+uu_x+u_x=0, \quad (29)$$

where $(x,t)\in[0,1]\times[0,T]$, with the boundary conditions

$$u(0,t)=u(1,t)=0, \quad u_{xx}(0,t)=u_{xx}(1,t)=0, \quad t\in[0,T], \quad (30)$$

and initial condition

$$u(x,0)=x^4(1-x^4), \qquad x\in[0,1], \quad (31)$$

We discredits the equations (29)-(31) using Quintic B-spline collocation method (6) with $k=\frac{1}{20}$, and $h=\frac{1}{5},\frac{1}{10},\frac{1}{20},\frac{1}{40}$. Since we do not know the exact solution of (29)-(31), a comparison between the numerical solutions on a coarse mesh and those on a refined mesh is made [13]. Since the numerical solution U_m of Quintic B-spline collocation method (8) is zero at boundaries $x=0,1$, we compute ratios of convergence at each time step n,

$$R_h^n=\frac{\left\|U_h^n-U_{\frac{h}{2}}^n\right\|+\left\|\Delta_h(U_h^n-U_{\frac{h}{2}}^n)\right\|}{\left\|U_{\frac{h}{2}}^n-U_{\frac{h}{4}}^n\right\|+\left\|\Delta_h(U_{\frac{h}{2}}^n-U_{\frac{h}{4}}^n)\right\|},$$

where $\Delta_h v_i^n=\frac{v_{i+1}^n-2v_i^n+v_{i-1}^n}{h^2}$.

The average ratio of convergence R_h^n, based on both infinite norm and L^2-norm $R_{av}=\frac{1}{21}\sum_{n=1}^{10}R_n$, on $0\le x\le 1$ and $0\le t\le 1$ are given in Tables 1 and 2. Here U_h^n is a numerical solution of (31) at $t_n=nk$ with step size h, which is shown in the Figs. 1, 2, 3 and Fig. 4.

(30)

Table 1. The ratios of convergence R_h^n, based on infinite norm when $k=\frac{1}{20}$

	R_h^n			
n	$h=\frac{1}{5}$	$h=\frac{1}{10}$	$h=\frac{1}{20}$	$h=\frac{1}{40}$
1	4.0966	4.0189	4.0042	4.0010
2	4.0966	4.0189	4.0042	4.0010
3	4.0982	4.0202	4.0050	4.0017
4	4.0997	4.0215	4.0058	4.0024
5	4.1012	4.0228	4.0066	4.0031
6	4.1027	4.0241	4.0073	4.0037
7	4.1042	4.0254	4.0081	4.0044
8	4.1057	4.0267	4.0089	4.0051
9	4.1072	4.0279	4.0097	4.0058
10	4.1088	4.0292	4.0104	4.0065
11	4.1103	4.0305	4.0112	4.0072
12	4.1118	4.0318	4.0120	4.0079
13	4.1133	4.0331	4.0128	4.0085
14	4.1148	4.0344	4.0136	4.0092
15	4.1163	4.0357	4.0143	4.0099
16	4.1178	4.0370	4.0151	4.0106
17	4.1193	4.0383	4.0159	4.0113
18	4.1208	4.0396	4.0167	4.0120
19	4.1223	4.0409	4.0175	4.0127
20	4.1239	4.0422	4.0182	4.0134
21	4.1254	4.0435	4.0190	4.0141
R_{av}	4.1103	4.0306	4.0113	4.0072

Table 2. The ratios of convergence R_h^n, based on L^2-norm when $k=\frac{1}{20}$

	R_h^n			
n	$h=\frac{1}{5}$	$h=\frac{1}{10}$	$h=\frac{1}{20}$	$h=\frac{1}{40}$
1	4.1041	4.0221	4.0052	4.0013
2	4.1041	4.0221	4.0052	4.0013
3	4.1059	4.0235	4.0057	4.0016
4	4.1077	4.0248	4.0062	4.0019
5	4.1095	4.0261	4.0068	4.0022
6	4.1113	4.0274	4.0073	4.0026
7	4.1131	4.0288	4.0078	4.0029
8	4.1149	4.0301	4.0083	4.0032
9	4.1167	4.0314	4.0089	4.0036
10	4.1185	4.0328	4.0094	4.0039
11	4.1203	4.0341	4.0099	4.0042
12	4.1220	4.0354	4.0105	4.0046
13	4.1238	4.0368	4.0110	4.0049

14	4.1256	4.0381	4.0115	4.0052
15	4.1274	4.0394	4.0120	4.0056
16	4.1292	4.0408	4.0126	4.0059
17	4.1310	4.0421	4.0131	4.0062
18	4.1328	4.0434	4.0136	4.0066
19	4.1345	4.0448	4.0142	4.0069
20	4.1363	4.0461	4.0147	4.0072
21	4.1381	4.0475	4.0152	4.0076
R_{av}	4.1203	4.0342	4.0100	4.0043

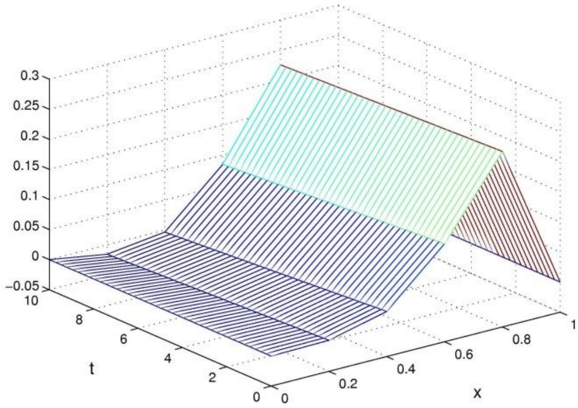

Fig. 1. The concentration numerical solution $U(x,t)$, for $h=\frac{1}{5}$ and $k=\frac{1}{20}$ plotted as a function of $x=0:h:1$ and $t=0:k:10$

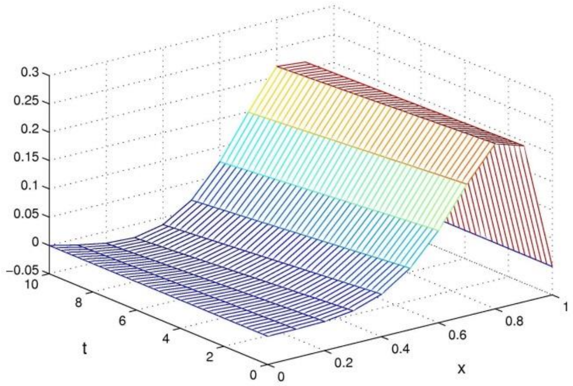

Fig. 2. The concentration numerical solution $U(x,t)$, for $h=\frac{1}{10}$ and $k=\frac{1}{20}$ plotted as a function of $x=0:h:1$ and $t=0:k:10$

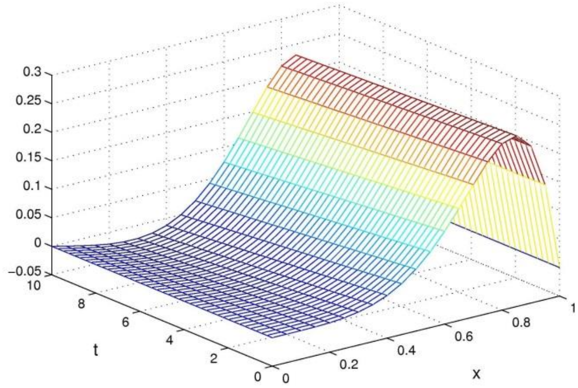

Fig. 3. The concentration numerical solution $U(x,t)$, for $h=\frac{1}{20}$ and $k=\frac{1}{20}$ plotted as a function of $x=0:h:1$ and $t=0:k:10$

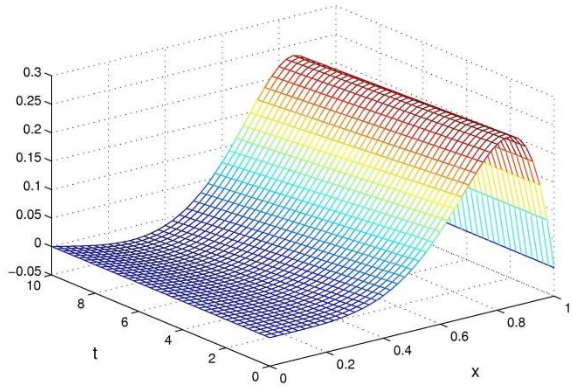

Fig. 4. The concentration numerical solution $U(x,t)$, for $h=\frac{1}{40}$ and $k=\frac{1}{20}$ plotted as a function of $x=0:h:1$ and $t=0:k:10$

6. Conclusions

In this paper, a numerical method for the nonlinear KdV-like Rosenau equation is proposed. This scheme is based on the Crank-Nicolson formulation for time integration and quintic B-spline functions for space integration. The structure, application and results of the employed method shows that the quintic B-spline method considered in this work is simple and straightforward. The employed method can be applied for a large class of linear and nonlinear problems. The obtained solution is presented graphically at various time steps which show the same characteristics as those given in the literature. Since we do not know the exact solution of the nonlinear KdV-like Rosenau equation, a comparison between the numerical solutions on a coarse mesh and those on a refined mesh is made. According to the ratios of convergence R_h^n, mentioned in the Tables 1 and 2, based on infinite norm and L^2-norm respectively, it can be concluded that the quintic B-spline collocation methods is both efficient and reliable for obtaining the numerical solutions of the partial differential equations.

Acknowledgments

We would like to express our sincere thanks and gratitude to the reviewer(s) for their valuable comments and suggestions for the improvement of this paper. Also, the authors would like to thank "Ardabil Branch, Islamic Azad university, Ardabil, Iran" for its financial support.

References

Chung, S. K. (1998), Finite difference approximate solutions for the Rosenau equation. *Appl. Anal*., 69(12), 149–156.

Chung, S. K., & Pani, A. K. (2001). Numerical methods for the Rosenau equation, *Appl. Anal*., 77, 351–369.

Dag, I., Saka, B., & Irk, D. (2006). Galerkin method for the numerical solution of the RLW equation using quintic B-splines. *Comput. Appl. Math*., 190, 532–547.

Gardner, G. A., Gardner L. R. T., & Ali, A. H. A. (1990). Modeling solitons of the Korteweg de Vries equation with quintic B-splines. *U.C.N.W. Math*., Preprint.

Kim, Y. D., & Lee, H. Y. (1998). The convergence of finite element Galerkin solution for the Rosenau equation. *Korean J. Comput. Appl. Math*., 5, 171–180.

Korteweg, D. J., & de Vries, G. (1895). On the Change of Form of Long Waves Advancing in a Rectangular Canal, and on a New Type of Long Stationary Waves. *Philosophical Magazine*, 39(240), 422–443.

Lee, H. Y., Ohm M. R., & Shin J. Y. (1997). Finite Element Galerkin approximations of the Rosenau equation. *Proceedings of Nonlinear Functional Analysis and Applications*, 2, 153–160.

Manickam, S. A., Pani, A. K., & Chung, S. K. (1998). A second-order splitting combined with orthogonal cubic spline collocation method for the Rosenau equation, *Numer. Meth. Partial Diff. Eq.*, 14, 695–716.

Park, M. A. (1990). On the Rosenau equation. *Math. Appl. Comput*., 9, 145–152.

Rosenau, P. (1986). A quasi-continuous description of a nonlinear transmission line, *Phys. Scripta*, 34, 827–829.

Rosenau, P. (1988). Dynamics of dense discrete systems. *Progr. Theor. Phys*., 79, 1028–1042.

Sepehrian, B., & Lashani, M. (2008). A numerical Solution of the Burgers equation using quintic B-splines. *In: Proceedings of the world congress on engineering* 2008, vol. III, WCE; UK: London.

Zaki, S. I. (2000). A quintic B-spline finite elements scheme for the KdVB equation. *Comput. Meth. Appl. Eng*., 188, 121–134.