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THE COHERENT ANOMALY METHOD IN THE STUDYING OF THE S=1 ISING CHAINS WITH LONG-RANGE INTERACTION [*]

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Abstract – The coherent anomaly method and a cluster mean-field approach combined with finite-size scaling extrapolation were used to determine the critical temperature and the critical exponents of a $S=1$ Ising chain with a long-range interaction in the form of $1/r^{1+\sigma}$. The results of critical temperature are in good agreement with the recent results of the finite-range scaling method and are more accurate in the classical region. The critical exponents are in agreement with the results of spin-$\frac{1}{2}$ Ising chains and indicate that the critical exponents are independent of the magnitude of spin in Ising models with long-range interactions.

Keywords – Coherent anomaly, critical phenomena, critical exponents, Ising model

1. INTRODUCTION

The spin Ising ferromagnet chains with a long-range interaction proportional to $1/r_{ij}^{1+\sigma}$ (r is the distance between spins at sites i and j) are known rigorously to exhibit long-range order for $0<\sigma<1$ [1]. There are different interests in the study of the critical phenomena in the systems with long-range interactions. A problem that is of particular interest is the $\sigma=1$ case. Under this form of interaction, the $S=1/2$ can be mapped onto the spin-$\frac{1}{2}$ Kondo problem [2] and generally, $S>1/2$ may be related to the higher spin generalization of the Kondo problem [3].

In the present work, a cluster mean-field approach combined with the finite-size scaling extrapolation [4] is used to determine critical temperature of a spin chain with a long-range interaction in the form of $1/r^{1+\sigma}$ for $S=1$. For determining the critical exponents of these chains, we employ the Coherent Anomaly Method (CAM) [5] with an appropriate extrapolation procedure based on Van den Broeck and Schwartz transformation (VBS) [6] and Least Squares Approximation (LSA).

2. COMPUTATIONAL METHOD

The Hamiltonian of the system under consideration can be written as

$$H_{\{s_i\}} = \sum_{i<j} J_{ij} s_i s_j - h \sum_i s_i \,, \tag{1}$$

where $s_i = -1,0,1$ and $J_{ij} = J/|i-j|^{1+\sigma}$ in which the lattice spacing is one unit and $\{s_i\}$ denotes a configuration of the system. The thermal average of a spin defined as

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$$\langle s_i \rangle = Z^{-1} \sum_{\{s_i\}} s_i \exp(-H(\{s_i\})/kT), \tag{2}$$

where Z is the partition function. Hereafter we set J/k equal to one.

Our approach in section 3 requires a sequence of critical temperature estimates for the above system in which $h=0$, and we achieve this by use of the cluster mean-field approach. In this approach, we specifically treat all interactions among the spins making up a cluster and we replace all interactions between a spin in the cluster and one outside the cluster with a mean-field interaction [4]. We denote this critical temperature as $T_c(L)$, with L representing the number of sites. Here we look at clusters with an odd number of sites. For the estimation of critical exponents we use the CAM [5]. Since our results are mean-field type, the spontaneous magnetization and the zero-field susceptibility are

$$m(L) = \overline{m}(L)|\varepsilon|^{1/2}, \chi(L) = \overline{\chi}(L)\frac{1}{\varepsilon}, \varepsilon = \frac{T - T_c(L)}{T_c(L)}. \tag{3}$$

The CAM method makes use of $\overline{m}(L)$ and $\overline{\chi}(L)$ to determine the true critical exponent values of β and γ. The values are given by

$$\beta = \frac{1}{2} - \frac{\log(\overline{m}(L_1)/\overline{m}(L_2))}{\log((T_c(L_2)-T_c)/(T_c(L_1)-T_c))}, \tag{4}$$

$$\gamma = 1 + \frac{\log(\overline{\chi}(L_1)/\overline{\chi}(L_2))}{\log((T_c(L_2)-T_c)/(T_c(L_1)-T_c))}, \tag{5}$$

where L_1 and L_2 denote two different cluster sizes and T_c is the true critical temperature for the infinite chain. L_1 and L_2 must be two closed integers for better convergency.

After obtaining the mean-field critical temperatures $T_c(L)$, we now use a finite-size approach to first get an approximation for the true critical temperature[4] and then we use Eqs(4) and (5) to obtain values for β and γ. We use the finite-size approach to again get the approximation for the true critical exponent β and γ.

From the scaling hypotheses one expects to observe a power-law convergency for the critical temperature and critical exponents [7]. For the sequence of A_L, the relation of convergency can be written as

$$A_L \cong A_\infty + b_1 L^{-\lambda_1} + L^{-\lambda_2} + \cdots, \tag{6}$$

where A_∞ is the value of A in the limit of $L \to \infty$. In this study we use two approaches to determine the value of A_∞. In the first approach we apply a procedure due to the Van den Broeck and Schwartz method (VBS) [6]. The procedure is a generalization of the Pade approximation method. Successive approximations are given by the following recurrence relations

$$\frac{1}{[L,N+1]-[L,N]} + \frac{[(-1)^N - 1]/2}{[L,N-1]-[L,N]} = \frac{1}{[L+1,N]-[L,N]} + \frac{1}{[L-1,N]-[L,N]}, \tag{7}$$

where $[L,N]$ is the N th-order extrapolation of A_L. In particular $[L,0] = A_L$ and $[L,-1] = \infty$.

In the second approach, we have used one simple extrapolation procedure, fitting the curve in the Least-Squares Approximation (LSA) to a power law form.

3. RESULTS AND DISCUSSION

The calculations of the mean-field critical temperature $T_c(L)$ by the cluster mean-field method are performed in the classical ($0 < \sigma \leq 0.5$) and nonclassical ($0.5 < \sigma \leq 1$) regions for S=1 Ising chains with a high numerical precision. For this purpose, the calculation was achieved using the Maple package. The mean-field critical temperatures were determined for $L = 1,3,\ldots,17$, and by the VBS method the true critical temperature was obtained. The significant digits of the data presented in this section was determined by the stable digits between the two last columns of VBS approximants. The results of critical temperatures in both the classical and nonclassical regions are presented in Table 1, and for comparison, we have shown the critical temperatures obtained by the Finite-Range Scaling method (FRS) [8, 9]. As is seen, the results of both methods are almost in good agreement, but as is expected, in the mean-field region our results are more accurate. It must be mentioned that the FRS method is an approach based on range-scaling in the nonclassical region, but our results based on the cluster mean-field are very near to the FRS results in the nonclassical region.

Table 1. The critical temperature estimates based on cluster mean-field approach and the VBS transformations and the results of FRS method in (a) the classical region and (b) nonclassical region

σ	0.1	0.2	0.3	0.4	0.5
VBS	14.0279	7.2819	4.9766	3.7806	3.0296
FRS [8]	14.0	7.32	4.99	3.78	3.026
FRS [9]	14.06	7.2801	4.9561	3.748	2.991

(a)

σ	0.6	0.7	0.8	0.9	1
VBS	2.5031	2.1062	1.7899	1.525	1.29
FRS [8]	2.499	2.103	1.789	1.530	1.317

(b)

After determination of the critical temperature, we use Eqs (4) and (5) to calculate estimates for β and γ respectively. It must be remarked that the accuracy of these estimates is lower because of the logarithmic form of these equations. Therefore, we use both the VBS and LSA methods in extrapolation. In the LSA method, the estimates of β and γ for $L = 11,13,15$ and 17 are fitted to the a power law form in the least-squares approximations. The results of the CAM method with both extrapolation procedures were presented in Tables 2 and 3, and the results of the other methods for spin-$\frac{1}{2}$ Ising chains also were shown for comparison.

Table 2. The β critical exponent values were obtained by CAM method with VBS transformations and LSA extrapolation. Other results are from various analytical and numerical methods for spin-1/2 Ising chains in (a) the classical region and (b) nonclassical region

σ	0.6	0.7	0.8	0.9	1
CAM with VBS	0.3640	0.3086	0.2504	0.2007	0.1541
CAM with LSA	0.36294	0.29385	0.2135	0.16417	0.13812
Ref [12]	0.381	0.354	0.328	0.299	0.265
Ref [13]	0.33	0.26	0.18	0.10	0.0

(a)

Table 2. (Continued)

σ	0.1	0.2	0.3	0.4	0.5
CAM with VBS	1.0006	1.0045	1.0172	1.045	1.090
CAM with LSA	1.0002	1.0007	0.999		
Ref [4]	1.0008	1.0060	1.023	1.064	1.137
Ref [10]	1.0	1.0	1.0	1.0	1.0
Ref [12]	1.014	1.040	1.097	1.167	1.264
Ref [13]		1.0	1.01	1.06	1.11

(b)

Table 3. The γ critical exponent values were obtained by CAM method with VBS transformations and LSA extrapolation. Other results are from various analytical and numerical methods for spin-1/2 Ising chains in (a) the classical region and (b) nonclassical region

σ	0.6	0.7	0.8	0.9	1
CAM with VBS	1.209	1.3686	1.490	1.65	1.84
Ref [10]	1.19	1.29	1.50	1.78	2.2
Ref [12]	1.360	1.463	1.584	1.701	1.846
Ref [13]	1.176	1.440	1.790	2.226	2.750

(a)

σ	0.1	0.2	0.3	0.4	0.5
CAM with VBS	0.4991	0.4939	0.4803	0.4545	0.4098
CAM with LSA	0.50001	0.50224			
Ref [4]	0.499507	0.49610	0.46458	0.43994	0.40843
Ref [10]	0.5	0.5	0.5	0.5	0.5
Ref [11]	0.494	0.482	0.497	0.51	0.51
Ref [12]	0.495	0.482	0.460	0.435	0.408
Ref [13]		0.5	0.48	0.45	0.39

(b)

As is seen, the critical exponents obtained in our approach for small values of σ in the classical region are in good agreement with the results of the RG method [10] and the extensive Monte Carlo method for spin-$\frac{1}{2}$ Ising chains [11]. In the nonclassical region, there are no reliable results for critical exponents, however there are some different results with low accuracy originating from the complicated nature of critical phenomena in this region. In this region, the comparison of our results and coherent anomaly results for spin-$\frac{1}{2}$ Ising chains [4, 12] indicates a relatively good agreement.

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