

“Research Note”

**SEMI-ANALYTICAL SOLUTION OF STABILITY OF COMPOSITE ORTHOTROPIC
CYLINDRICAL SHELLS UNDERTIME DEPENDENT A-PERIODIC
AXIALCOMPRESSIVE LOAD***

H. SOFIYEV^{1**}, AL. H. SOFIYEV², E. YUSUFOGLU³, AND Z. KARACA⁴

¹Department of Civil Engineering, Suleyman Demirel University, Isparta, Turkey
Email: asofiyev@mmf.sdu.edu.tr

²Technical Sciences Department, Kazakh Branch, Azerbaijan Teachers Institute, Kazakh, Azerbaijan

³Department of Mathematics, Dumlupinar University, Kutahya, Turkey

⁴Department of Civil Engineering, Ondokuz Mayıs University, Samsun, Turkey

Abstract – In this study, the stability problem of a circular orthotropic cylindrical shell under the effect of an axial compression varying with a power function of time is considered. At first, the modified Donnell type dynamic stability and compatibility equations are obtained using Love’s shell theory. Applying the Galerkin method and Rayleigh-Ritz variational techniques to these equations and taking the large values of loading parameters into consideration, analytics are obtained for critical parameter values. The results show that critical parameters are affected by loading parameters variations, ratio of the Young’s moduli variations, radius to thickness variations and the power of time in the axial compression expression variations. Comparing results with those in the literature validates the present analysis.

Keywords – orthotropic material, cylindrical shell, dynamic critical axial load, dynamic factor

1. INTRODUCTION

The thin cylinder under axial compression is a fundamental problem in shell theory mainly due to its wide use as a structural element in several engineering areas: mechanical, aeronautical, and nuclear. The study of the stability of cylindrical shells under dynamic load is an important aspect in the successful applications of the shell. It is known that the researchers use two concepts in the solution of dynamic stability problems.

The first concept is applied when the stability problem of construction under sudden loadings are affected in a very short time (impact). By using this concept, dynamic buckling problems of cylindrical shells under dynamic axial loads are solved experimentally, numerically and by using the finite element method [1, 2].

The second concept is based on the assumption that, stress and deformation occurring in different points of the deformable body under the effect of dynamic load, propagates suddenly to the whole volume of the body. When the equation of the motion of the system (shell) element is constituted, an inertia force corresponding to the normal displacement is taken into consideration. Consequently, in this state, propagation of elastic waves in the middle surface is not taken into consideration. By using this concept, many dynamics problems of cylindrical shells under sudden loads are solved experimentally, analytically and by numerical integration [3-5]. The fundamental work concurring this concept is Agamirov [6].

*Received by the editor July 12, 2005 and in final revised form October 18, 2006

**Corresponding author

In recent years, the buckling problems of shells under external pressure and torsion dependent on time are studied in [6, 7]. The analytical solutions of the stability of the shells under the axial compression, varying with a power function of time, have not been considered to date.

In this paper, by applying the second concept and Rayleigh-Ritz variational techniques, the stability analysis of a circular orthotropic cylindrical shell under the effect of an axial compression varying with a power function of time is studied.

2. FORMULATION OF THE PROBLEM

Assume that a circular cylindrical shell with radius R , length L and thickness h is subjected to uniformly distributed axial loads,

$$N_{x0} = -(T_1 + T_0 t^q) \quad (1)$$

along both edges (Fig. 1). Where N_{x0} is the membrane force for the condition with zero initial moments, T_1 is the static axial compression, T_0 is the axial loading parameter, t is time and $q \geq 1$ is the power expressing the time dependence of the load.

The orthogonal co-ordinate system is fixed on the middle surface of the shell. The x -axis is taken along a generator and y -axis is taken as tangential directions, with the z axis normal to them. The axes of orthotropy are parallel to the x and y -axes (Fig. 1).

According to the shell theory, after lengthy computations, the stability and compatibility equations of orthotropic cylindrical shells under axial load, which is a power function of time, are obtained as follows [6, 8]:

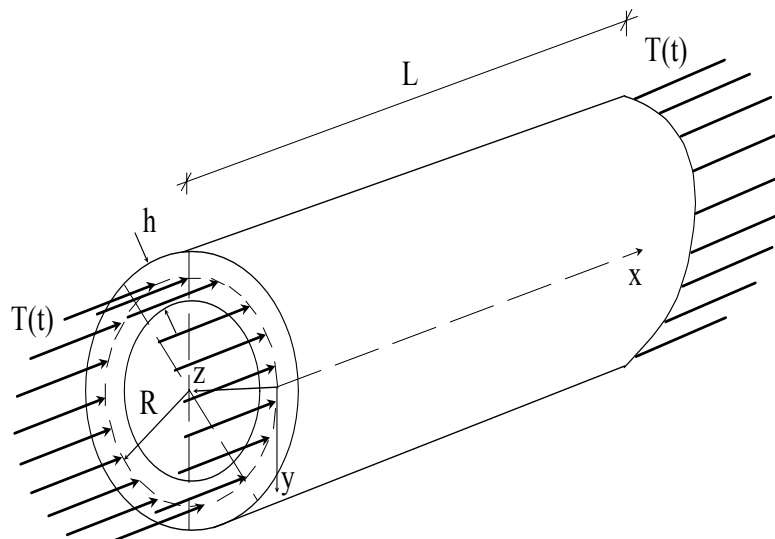


Fig 1. Geometry and the coordinate system of a cylindrical thin shell

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\mu_{yx}^{1/2}}{\mu_{xy}^{1/2}} \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\mu_{yx}}{\mu_{xy}} \frac{\partial^4 w}{\partial y^4} - \frac{12(1 - \mu_{xy}\mu_{yx})}{E_x h^3} \left[\frac{1}{R} \frac{\partial^2 \Phi}{\partial x^2} - (T_1 + T_0 t^q) \frac{\partial^2 w}{\partial x^2} - \rho h \frac{\partial^2 w}{\partial t^2} \right] = 0 \quad (2)$$

$$\frac{\partial^4 \Phi}{\partial x^4} + 2 \frac{\mu_{xy}^{1/2}}{\mu_{yx}^{1/2}} \frac{\partial^4 \Phi}{\partial x^2 \partial y^2} + \frac{\mu_{xy}}{\mu_{yx}} \frac{\partial^4 \Phi}{\partial y^4} + \frac{E_y h}{R} \frac{\partial^2 w}{\partial x^2} = 0 \quad (3)$$

3. SOLUTIONS OF THE DIFFERENTIAL EQUATIONS

Assuming the cylindrical shell to have hinged supports at the ends, the solution of the equation set (2)-(3) is sought in the following form [6]:

$$w = \xi(t) \sin \frac{m_1 x}{R} \cos \frac{ny}{R}, \quad \Phi = \zeta(t) \sin \frac{m_1 x}{R} \cos \frac{ny}{R} \quad (4)$$

where $m_1 = m\pi R/L$, m is the half wave number in the direction of the x axis, n is the wave number in the direction of the y axis and $\xi(t)$ and $\zeta(t)$ are the amplitudes.

Substituting expressions (4) in the equation set (2)-(3), then applying Galerkin's method in the ranges $0 \leq x \leq L$ and $0 \leq y \leq 2\pi R$ and eliminating $\zeta(t)$, the following differential equation is obtained:

$$\frac{d^2 \xi(\tau)}{d\tau^2} + (\lambda_1 - \lambda_0 \tau^q) \xi(\tau) = 0 \quad (5)$$

where $t = \tau t_{cr}$, in which t_{cr} is the critical time and the dimensionless time parameter τ satisfies $0 \leq \tau \leq 1$; $\lambda_i (i = 1, 2)$ are parameters depending on the properties of the orthotropic material and the characteristics of the cylindrical shell.

An approximating function will be chosen as $\xi(\tau) = A e^{\alpha \tau} \tau^2 [(\alpha + 3)/(\alpha + 2) - \tau]$, satisfying the initial conditions $\xi(0) = \xi_{,\tau}(0) = 0$. Here A is constant. The values of α will be determined after the formula of the dynamic critical axial load is found.

Let us consider the axisymmetric stability loss ($n=0$). Applying Rayleigh-Ritz variational techniques to Eq. (5), after some mathematical operations, when the axial compression varies linearly and parabolically dependent on time, the following expressions are found for the static critical axial load, dynamic critical axial load and dynamic factor (D_f), respectively:

$$T_{st}^{cr} = \frac{h}{\sqrt{3}R} \left(\frac{E_x E_y}{1 - \mu_{xy} \mu_{yx}} \right)^{1/2} \quad (6)$$

$$T_d^{cr}(1) = T_0 t_{cr} / h = \frac{B_0(1) E_x h^2}{6(1 - \mu_{xy} \mu_{yx}) R^2} \left[0.5 \gamma_1 + 0.5 (\gamma_1^2 + 4 \gamma_3) \right]^{1/2} \quad (7)$$

$$T_d^{cr}(2) = T_0 t_{cr}^2 / h = \frac{B_0(2) E_x h^2}{6(1 - \mu_{xy} \mu_{yx}) R^2} \gamma_4 \quad (8)$$

$$D_f(1) = T_d^{cr}(1) / T_{st}^{cr}; D_f(2) = T_d^{cr}(2) / T_{st}^{cr} \quad (9-10)$$

where $B_i(q)$, $i=0,1$ given in [7], and $\gamma_i (i = 1 \div 4)$ are parameters depending on the properties of the orthotropic material and the characteristics of the cylindrical shell.

Expressions (7) and (8) can be written in dimensionless form as in the following:

$$\hat{T}_d^{cr}(1) = T_d^{cr}(1) R / (h \sqrt{E_x E_y})^{1/2}, \quad \hat{T}_d^{cr}(2) = T_d^{cr}(2) R / (h \sqrt{E_x E_y}) \gamma_4 \quad (11-12)$$

4. NUMERICAL COMPUTATIONS AND RESULTS

To verify the numerical results obtained in this study, the values of the dynamic critical axial load values found by the present method, when the axial compression varies linearly dependent on time are compared in Table 1 with the dynamic critical axial load values obtained by numerical integration (Runge-Kutta method), given in [3]. The computations presented in Table 1 have been carried out for the following

material properties and shell parameters given in Shumik [3], $E_x=10^4$ MPa, $E_y=2 \times 10^4$ MPa, $\mu_{xy}=\mu_{yx}=0.15$, $\rho=1.83 \times 10^3$ kg/m³, $q=1$, $L/R=2$, $R/h=100$, $\bar{\xi}_0 = 0.001$. There is a good agreement between the present and the numerical results.

In Table 2 the variations of the dynamic critical axial loads, the corresponding values of the wave number (m_d), and the dynamic factor with the ratios of Young's module and R/h are seen. In parenthesis are the values of α corresponding to the minimum values of the dynamic critical axial load.

When $E_x (=1.724 \times 10^5$ MPa) is kept constant and the ratio E_x/E_y is increased, the values of the dynamic critical axial load and corresponding wave numbers decrease, whereas the values of the dynamic factor increases.

When $E_x (=7.79 \times 10^3$ MPa) is kept constant and the ratio E_y/E_x is increased, the values of the dynamic critical axial load and corresponding wave numbers increases, whereas the values of the dynamic factor decrease. Hence, materials with high degrees of anisotropy can cause the loss of stability of cylindrical shells.

When the ratio R/h increases, the values of the dynamic critical axial load decreases significantly, whereas the values of the dynamic factor increases significantly. When the ratio R/h increases, the values of the wave number corresponding to dynamic critical axial load increase, while those values of α decrease.

Table 1. Comparisons of the dynamic critical axial load with numerical results for $q=1$

Shumik [3] (Runge-Kutta method)		Present study	
$(T_0/h) \times 10^{-5}$ (MPa/sec)	$T_d^{cr}(1)$ (MPa)	$T_d^{cr}(1)$ (MPa)	α
0.2	85.0	86.6	57
0.4	91.7	89.0	37
5.0	160.6	160.8	1.66

Table 2. Variation of the critical parameters with the ratio of Young's moduli and $R/h(T_0/h=10^5$ MPa/s, $L/R=2$)

E_x/E_y	$R/h=100$			$R/h=200$			$R/h=300$		
	$T_d^{cr}(1)$ (MPa)	$D_f(1)$	m_d	$T_d^{cr}(1)$ (MPa)	$D_f(1)$	m_d	$T_d^{cr}(1)$ (MPa)	$D_f(1)$	m_d
10	335.0 ($\alpha=49$)	1.058	6	181.3 ($\alpha=19$)	1.145	9	131.6 ($\alpha=1$)	1.25	12
20	243.7 ($\alpha=31$)	1.092	5	137.2 ($\alpha=14$)	1.229	8	103.4 ($\alpha=9$)	1.39	10
30	203.9 ($\alpha=24$)	1.12	4	118.3 ($\alpha=11$)	1.299	7	91.4 ($\alpha=7$)	1.51	9
E_y/E_x	$T_d^{cr}(1)$ (MPa)	$D_f(1)$	m_d	$T_d^{cr}(1)$ (MPa)	$D_f(1)$	m_d	$T_d^{cr}(1)$ (MPa)	$D_f(1)$	m_d
10	150.9 ($\alpha=47$)	1.06	21	81.83 ($\alpha=20$)	1.149	30	59.5 ($\alpha=12$)	1.25	37
20	209.2 ($\alpha=74$)	1.037	25	110.3 ($\alpha=31$)	1.094	35	78.0 ($\alpha=19$)	1.16	44
30	254.4 ($\alpha=97$)	1.029	27	132.5 ($\alpha=39$)	1.072	39	92.6 ($\alpha=24$)	1.12	48

5. CONCLUSIONS

In the present research, the stability of orthotropic cylindrical thin shells under axial compression, a power function of time, was studied. At first, the fundamental relations and modified Donnell type dynamic stability equations have been written. Then, applying Galerkin's method, a time dependent differential equation with a variable coefficient has been obtained. Finally, the critical parameters are found analytically for an axially symmetry case by applying the Rayleigh-Ritz variational techniques. The effects of the variations of the loading parameter, the ratio of radius to thickness, the ratio of Young's

moduli and the power of time in the axial compressive load expression on critical parameters, have been studied numerically.

REFERENCES

1. Jones, N. (1997) Structural Impact. *Cambridge University Press, Cambridge 1989, Paperbacked.*
2. Jones, N., Karagiozova, D. & Alves, M. (2000). Inertia effects in axisymmetrically deformed cylindrical shells under axial impact. *International Journal of Impact Engineering*, 24, 1083-1115.
3. Shumik, M. A. (1972). The buckling of fiberglass cylindrical shell subjected dynamic axial load. *Mechanics of Polymers*, 4, 740-743.
4. Tazyukov, F. Kh. (1991). The stability of plates and shells under impulsive loading (in Russian). Research on the Theory of Plates and Shells, *Kazan State University*, 23, 119-130.
5. Meish, V. F. (1996). Dynamic behavior of a rectangular orthotropic plate subjected to a sudden load. *International Applied Mechanics*, 32(3), 219-224.
6. Agamirov, V. L. (1990). *Dynamic Problems of Nonlinear Shells Theory*. Nauka, Moscow (in Russian).
7. Sofiyev, A. H. (2003). Torsional buckling of cross-ply laminated orthotropic composite cylindrical shells subject to dynamic loading. *European Journal of Mechanics A/Solids* 22, 943-951.
8. Vinson, J. R. & Sierakowski, R. L. (1986). The Behavior of Structures Composed of Composite Material. *Nijhoff, Dordrecht.*