IJST (2014) 38A4: 473-479
Iranian Journal of Science & Technology

http://ijsts.shirazu.ac.ir

Numerical technique for integro-differential equations arising in oscillating magnetic fields

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Abstract

In this paper, we propose the Chebyshev wavelet approximation for the numerical solution of a class of integro-differential equation which describes the charged particle motion for certain configurations of oscillating magnetic fields. We show that the Chebyshev approximation transform an integral equation to an explicit system of linear algebraic equations. Illustrative examples are included to demonstrate the validity and applicability of the new technique.

Keywords*:* Integro-differential equation; Chebyshev wavelet; charged particle motion; oscillating magnetic field

1. Introduction

In the recent years, there has been increased usage among scientists and engineers to apply wavelet technique to solve both linear and nonlinear problems (Razzaghi et al., 2002; Lashab et al., 2008; Lashab et al., 2007; Tretiakov and Pan, 2004a; Tretiakov and Pan, 2004 b; Yan et al., 2014; Jiao et al., 2014; Bakar et al., 2014; Perez-Munoz, 2013) and their references. The main advantage of the wavelet technique is its ability to transform complex problems into a system of algebraic equations. The overview of this method can be found in (Razzaghi and Yousefi, 2002; Razzaghi and Shamsi, 2004; Ghasemi et al., 2007; Tavassoli Kajani et al., 2006; Tavassoli Kajani et al., 2009; Tavassoli Kajmani and Ghasemi, 2009; Ghasemi and Tavassoli Kajani, 2011; Heydari et al., 2012). In this work, we use Chebyshev wavelets on the interval [0, 1) to solve the integro-differential equation arising in oscillating magnetic fields. We clarify how the Chebyshev approximation transforms an integral equation to an explicit system of linear algebraic equations. We then apply the method to some numerical tests to clarify the efficiency of the method. Integral and integro-differential equations of Volterra type arise in many modeling problems in physical fields such as optics, electromagnetism, electrodynamics, statistical physics, inverse scattering problems and many other practical applications. In this research, we study an integro-differential equation which describes the charged particle motion for certain configurations of oscillating magnetic fields. Consider the following

Volterra integro-differential equation (Dehghan et al., 2012):

$$\frac{d^2y}{dt^2} = -a(t)y(t) + b(t) \tag{1}$$

where a(t), b(t) and g(t) are given periodic functions of time which may be easily found in the charged particle dynamics for some field configurations. y(t) is an unknown function to be determined. The existence and the uniqueness results of these types of problems have been investigated by many authors. For instance, using Theorem 4 in (Bojedain, 1991), the existence and the uniqueness issues of the second kind of integro-differential equation (1) can be deduced. Throughout this paper, we assumed that the conditions of the given functions of the Eq. (1) are such that the existence and the uniqueness results of the solution of Eq. (1) are satisfied. For clarifying the model, suppose that the three mutually orthogonal magnetic field components are defined as:

$$B_x = B_1 \sin(w_p t), \quad B_y = 0, \ B_z = B_0 .$$

So, the nonrelativistic equations of motion for a particle of mass m and charge q in this field configuration are:

$$m\frac{d^2x}{dt^2} = q\left(B_0 \frac{dy}{dt}\right), \tag{2}$$

$$m\frac{d^2y}{dt^2} = q\left(B_1 \sin(\mathrm{w_p t})\frac{dz}{dt} - B_0 \frac{dx}{dt}\right), \tag{3}$$

Received: 26 July 2013 / Accepted: 11 January 2014

$$m\frac{d^2z}{dt^2} = q\left(-B_1 \sin(\mathrm{w_p t})\frac{dy}{dt}\right). \quad (4)$$

By integration of Eq. (2) and Eq. (4) and replacing the time first derivatives of z and x in Eq (3) one achieves Eq. (1) with:

$$a(t) = w_c^2 + w_f^2 sin^2(w_p t),\ b(t) = w_f^2 w_p \sin(w_p t), \quad (5)$$

$$g(t) = w_f (\sin(w_p t))\, z'(0) + w_c^2 y(0) + w_c x'(0). \quad (6)$$

where $w_c = q\frac{B_0}{m}$ and $w_f = q\frac{B_1}{m}$. By the additional simplifications and seting $x''(0)$ =0 and $y(0) = 0$, the Eq. (1) can finally be written as:

$$\frac{d^2y}{dt^2} = -w_c^2 - w_f^2 sin^2(w_p t)y - w_f (\sin(w_p t))\, z'(0) \quad (7)$$
$$+ w_f^2 w_p sin\ (w_p t) + \int_0^{\mathrm{t}} \cos(\mathrm{w_p s})\mathrm{y(s)ds}.$$

The numerical solvability of Eq. (1) and the other related equations have been pursued by several authors. Dehghan and Shakeri (see (Dehghan et al., 2008), applied the homotopy perturbation method for solving Eq. (1). Machado et al. in (Machado and Tsuchida, 2002) solved Eq. (1) by using Adomian's method.

2. Properties of Chebyshev wavelets

2.1. Wavelets and Chebyshev wavelets

Wavelets constitute a family of functions constructed from dilation and translation of a single function called the mother wavelet. When the dilation parameter a and the translation parameter b vary continuously, we have the following family of continuous wavelets as (Ghasemi and Tavassoli Kajani, 2011):

$$\psi_{a,b}(t) = |a|^{-\frac{1}{2}}\psi\left(\frac{t-b}{a}\right), a, b \in R, \qquad a \neq 0.$$

Chebyshev wavelets $\psi_{nm}(t) = \psi(n, m, t)$ have three arguments, namely: n = 1, 2...2 k, k ∈ Z^+, m is the order for Chebyshev polynomials, and t is the normalized time. The Chebyshev wavelets are defined on the interval [0, 1):

$$\psi_{nm}(t) = \begin{cases} \frac{\alpha_m 2^{\frac{k}{2}}}{\sqrt{\pi}} T_m(2^{k+1}t - 2n + 1), & \frac{n-1}{2^k} \leq t \leq \frac{n}{2^k}, \\ 0 & otherwise, \end{cases}$$

where:

$$\alpha_m = \begin{cases} \sqrt{2} & m = 0, \\ 2 & m = 1,2 \ldots. \end{cases}$$

Here $T_m(t)$ are the well-known Chebyshev polynomials of order m, which are orthogonal with respect to the weight function $w(t) = \frac{1}{\sqrt{1-t^2}}$ and satisfy the following recursive formula:

$$T_0(t) = 1,$$
$$T_1(t) = 1,$$
$$T_{m+1}(t) = 2tT_m(t) - T_{m-1}(t).\ m = 1, 2,3, \ldots.$$

The set of Chebyshev wavelets are an orthonormal set with respect to the weight function $\widetilde{w}(t) = w(2^{k+1}t - 2n + 1)$.

2.2. Function approximation

A function $f(t)$ defined over [0, 1) may be expanded as:

$$f(t) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} f_{mn}\,\psi_{nm}(t), \quad (8)$$

where:

$$f_{nm} = \langle f(t), \psi_{nm}(t)\rangle \quad (9)$$

In Eq. (9), the symbol $\langle \ldots \rangle$ denotes the inner product with weight function $\widetilde{w}(t)$. If the infinite series in Eq. (8) is truncated, then Eq. (8) can be written as:

$$f(t) \cong \sum_{n=1}^{2^k}\sum_{m=0}^{M-1} f_{nm}\,\psi_{nm}(t) = F^T\psi(t), \quad (10)$$

where F and ψ(t) are $2^k M \times 1$ matrices given by:

$$F = \left[f_{1,0}, f_{1,1}, \ldots, f_{1,M-1}, f_{2,0}, f_{2,1}, \ldots, f_{2,M-1}, \ldots, f_{2^k,0}, f_{2^k,1}, \ldots, f_{2^k,M-1}\right]^T, \quad (11)$$

$$\Psi(\mathrm{t}) = \left[\psi_{1,0}(t), \ldots, \psi_{1,M-1}(t), \psi_{2,0}(t), \ldots, \psi_{2,M-1}(t), \ldots, \psi_{2^k,0}(t), \ldots, \psi_{2^k,M-1}\right]^T. \quad (12)$$

2.3. Chebyshev wavelets operational matrix of integration

The integration of the vector Ψ(t) which is defined in Eq. (12) can be obtained as:

$$\int_0^t \Psi(\mathrm{s})\, ds \simeq P\Psi(\mathrm{t}). \quad (13)$$

where P is the $(2^k M) \times (2^k M)$ operational matrix for integration and is given in (Ghasemi and Tavassoli Kajani, 2011) as:

$$P=\begin{pmatrix} C & S & S & \cdots & S\\ 0 & C & S & \cdots & S\\ 0 & C & C & \cdots & S\\ \vdots & \vdots & \vdots & \ddots & S\\ 0 & 0 & 0 & \cdots & C\end{pmatrix}$$

where S and C are $M\times M$ matrices given by:

$$S=\frac{\sqrt{2}}{2^k}\begin{bmatrix} 1 & 0 & 0 & \cdots & 0\\ 0 & 0 & 0 & \cdots & 0\\ -\frac{1}{3} & 0 & 0 & \cdots & 0\\ -\frac{1}{15} & 0 & 0 & \cdots & 0\\ \vdots & 0 & 0 & \ddots & \vdots\\ -\frac{1}{M(M-2)} & 0 & 0 & \cdots & 0\end{bmatrix}$$

and

$$C=\frac{1}{2^k}\begin{bmatrix} \frac{1}{2} & \frac{1}{2\sqrt{2}} & 0 & 0 & \cdots & 0 & 0 & 0\\ -\frac{1}{8\sqrt{2}} & 0 & \frac{1}{8} & 0 & \cdots & 0 & 0 & 0\\ -\frac{1}{6\sqrt{2}} & -\frac{1}{4} & 0 & \frac{1}{12} & \cdots & 0 & 0 & 0\\ \vdots & \vdots & \vdots & & \ddots & \vdots & \vdots & \vdots\\ -\frac{1}{2\sqrt{2}(M-1)(M-3)} & 0 & 0 & 0 & \cdots & -\frac{1}{4(M-3)} & 0 & -\frac{1}{4(M-2)}\\ -\frac{1}{2\sqrt{2}M(M-2)} & 0 & 0 & 0 & \cdots & 0 & -\frac{1}{4(M-2)} & 0\end{bmatrix}.$$

The integration of the product of two Chebyshev wavelet vector functions is obtained as:

$$D=\int_0^1\Psi(t)\Psi^T(t)dt\simeq\begin{bmatrix} D_1 & 0 & \cdots & 0\\ 0 & D_1 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & D_1\end{bmatrix},$$

where D is a symmetric matrix and D_1 is defined as follows:

$$(D_1)_{ij}=\frac{\alpha_i\alpha_j}{2\pi}\int_1^{-1}T_i(t)T_j(t)dt\,.$$

3. Applying Chebyshev wavelet to the problem

In this section, we use the Chebyshev wavelet to approximate a given function. Then by substituting these approximations in the linear integro-differential equation and using the collocation points, the equation will be transformed into a system of algebraic equations.

3.1. Oscillating magnetic field integro-differential equations

We consider the Eq. (1) with the following initial conditions:

$$g(0)=\alpha\,,\ y'(0)=\beta\,. \qquad (14)$$

Second order derivation of the function y(t) in Eq. (1) exists, so:

$$y(t)=\int_0^t\left(\int_0^x y''(s)+y'(0)\right)dx+y(0)\,. \qquad (15)$$

Approximating the functions y (s) and y ″ (s) with respect to the basis functions by Eq. (10) gives:

$$y(s)\approx Y^T\Psi(s),\quad y''(s)\approx Y''^T\Psi(s)\,. \qquad (16)$$

Substituting Eq. (16) into Eq. (15) and using Eq. (13), we obtain:

$$Y^T\Psi(t)\approx Y''^T P^2\,\Psi(t)+ty'(0)+y(0) \qquad (17)$$

In Eq. (17), two functions ty ′ (0) and y (0) can be approximated as:

$$ty'(0)\approx H^T\Psi(t)\,,\ y(0)\approx K^T\Psi(t). \qquad (18)$$

Therefore, we get:

$$Y^T\approx Y''^T P^2+H^T+K^T. \qquad (19)$$

Combining Eq. (1) and Eq. (17) yields:

$$Y''^T\left(\Psi(t)+a(t)P^2\,\Psi(t)-b(t)P^2\int_0^t cos(\mathrm{w_p}s)\Psi(s)ds\right)$$

$$= g(t) - a(t)\big(ty'(0) + y(0)\big) + b(t)\int_0^t cos(\mathrm{w_p s})\,(sy'(0) + y(0))\,ds.$$

Now, let $t_i = \frac{2i-1}{2^{M+3}}$, $i = 1, 2, \cdots, 2^{M+2}$ be 2^{M+2} collocation points in interval [0, 1). Substituting $t = t_i$ into Eq. (20), we have a linear system of 2^{M+2} algebraic equations of 2^{M+2} unknown coefficients corresponding to $y''(t)$. Solving this system of algebraic equations and substituting the results into Eq. (19) determines Y^T.

4. Error and convergence analysis

In this section, a convergence analysis is given for the proposed method. It is well-know that Chebyshev wavelets ψ_{nm} (t) forms a complete $L^2_{\widetilde{w}}([0,1])$ orthogonal set (Caunto et al., 1988), where $L^2_{\widetilde{w}}([0,1])$ denotes the space of all functions u: [0, 1] → ℜ with weighted L^2 -norm and define by:

$$\|u\|_{L^2_{\widetilde{w}}([0,1])^2_{\widetilde{w}}} = \int_0^1 u^2(t)\widetilde{w}(t)dt\,.$$

We recall that $H^m_{\widetilde{w}}([0,1])$ is the **Sobolev space** of all functions u(t) on [0, 1] such that its weak derivatives up to order m are in $L^2_{\widetilde{w}}([0,1])$ and define $\|.\|_{H^m_{\widetilde{w}}([0,1])}$as:

$$\|u\|_{H^m_{\widetilde{w}}([0,1])} = \left(\sum_{n=0}^{m}\left\|\frac{\partial^n}{\partial t^n}u(t)\right\|^2_{L^2_{\widetilde{w}}([0,1])}\right)^{\frac{1}{2}}.$$

Now, suppose that $u_{2^k,M-1}$ is the Chebyshev approximation of a function $u \in H^m_{\widetilde{w}}([0,1])$, then, the truncation error is (Canuto, 1988):

$$\|u - \mathbf{u}_{2^k,M-1}\|_{H^m_{\widetilde{w}}([0,1])} \le CN^{-m}\max_{0\le t\le 2^k}\|u\|_{H^m_{\widetilde{w}}(I_n)}, \quad (21)$$

where, $N = 2^kM - 1$, $I_n = \left[\frac{n-1}{2^k}, \frac{n}{2^k}\right]$ and C is a positive constant independent of m.

We note that, for large values of N, accuracy of the solution will be improved.

Now, consider Eq. (1) and let y be the exact solution of this equation. Let the approximate solution of y be y_N and the error defined by $e_N(t) = y(t) - y_N(t)$. Therefore, from Eq. (1) we get:

$$D^2 e_N(t) + a(t)e_N(t) - b(t)\int_0^t cos(\mathrm{w_p s})e_N(s)ds = e_{gN}(t), \quad (22)$$

where, $D^2 = \frac{d^2}{dt^2}$ and $e_{gN} = g - g_N$ with:

$$g_N(t) = D^2 y_N(t) + a(t)y_N(t) - b(t)\int_0^t cos(\mathrm{w_p s})y_N(s)ds. \quad (23)$$

Taking the norm $H^m_{\widetilde{w}}([0,1])$ from both sides of Eq. (22) yields:

$$\|e_{gN}\|_{H^m_{\widetilde{w}}([0,1])} \le \|D^2 e_N(t)\|_{H^m_{\widetilde{w}}([0,1])} + K_a\|e_{gN}\|_{H^m_{\widetilde{w}}([0,1])} + K_b\|e_{gN}\|_{H^m_{\widetilde{w}}([0,1])}, \quad (24)$$

where $K_{a(or\ b)} = \max_{0\le t\le 2^k}\|a(t)(or\ b(t))\|_{H^m_{\widetilde{w}}([0,1])} < \infty$. There exists a constant $\bar{C}$ such that $\|D^2{}_e\|_{H^m_{\widetilde{w}}([0,1])} \le \bar{C}\|e\|_{H^m_{\widetilde{w}}([0,1])}$. (Canuto, 1988). Now from this and Eq. (24) we find:

$$\|e_{gN}\|_{H^m_{\widetilde{w}}([0,1])} \le (\bar{C} + K_a + K_b)\|e_N\|_{H^m_{\widetilde{w}}([0,1]}. \quad (25)$$

Finally, combining Eq. (21) and Eq. (25) gives the following bound for the error in approximate solution:

$$\|e_{gN}\|_{H^m_{\widetilde{w}}([0,1])} \le (\bar{C} + K_a + K_b)CN^{-m}\max_{0\le t\le 2^k}\|y(t)\|_{H^m_{\widetilde{w}}(I_n)}. \quad (26)$$

Note that, since [0, 1] is a compact set, $\|y(t)\|_{H^m_{\widetilde{w}}(I_n)}$ is bounded and this means that the approximation is convergent for sufficiently large value **N**.

Theorem 1. The series solution (10) of Eq. (1) using Chebyshev wavelet method converges toward $y(t)$ as $M \to \infty$.

Proof: For k=0, the Eq. (10) reads as $y(t) \simeq \sum_{m=0}^{M-1} y_{1m}\psi_{1m}(t)$, where $y_{1m} = \langle y(t), \psi_{1m}(t)\rangle$. For simplicity, we denote $\psi_{1m}(t)$ as $\psi_m(t)$ and $\gamma_j = \langle y(t), \psi_j(t)\rangle$. Let $\{S_N(=\sum_{j=0}^{N}\gamma_j\psi_j(t))\}_{N=0}^{\infty}$ be the sequence of partial sums of $\sum_{j=0}^{\infty}\gamma_j\psi_j(t)$, then we have $\langle y(t), S_N\rangle = \langle y(t), \sum_{j=0}^{N}\gamma_j\psi_j(t)\rangle =$

$\sum_{j=0}^{N} \bar{\gamma}_j \langle y(t), \psi_j(t) \rangle = \sum_{j=0}^{N} |\gamma_j|^2$.

Now, let S_{N_1} and S_{N_2} be arbitrary partial sums with $N_1 \geq N_2$. We assert that $\|S_{N_1} - S_{N_2}\|^2 = \sum_{j=N_{2+1}}^{N_1} |\gamma_j|^2$ for $N_1 \geq N_2$. For this, we have $\left\|\sum_{j=N_{2+1}}^{N_1} \gamma_j \psi_j(t)\right\|^2 = \langle \sum_{i=N_{2+1}}^{N_1} \gamma_i \psi_i(t), \sum_{j=N_{2+1}}^{N_1} \gamma_j \psi_j(t) \rangle = \sum_{i=N_{2+1}}^{N_1} \sum_{j=N_{2+1}}^{N_1} \gamma_i \bar{\gamma}_i \langle \psi_i(t), \psi_j(t) \rangle = \sum_{j=N_{2+1}}^{N_1} |\gamma_j|^2$.

According to the Bessels inequality, we have $\sum_{j=0}^{\infty} |\gamma_j|^2$ which is convergent and $\|S_{N_1} - S_{N_2}\|^2 \to 0$, so $\|S_{N_1} - S_{N_2}\| \to 0$ as $N_1, N_2 \to \infty$. Therefore, we proved that S_N is a Cauchy sequence in Hilbert space, so then it is convergent.

Now, we claim that $(t) = S := \lim_{N\to\infty} S_N$. We have, $\langle y(t) - S, \psi_j(t) \rangle = \langle y(t), \psi_j(t) \rangle - \langle S, \psi_j(t) \rangle = \gamma_j - \langle \lim_{N\to\infty} S_N, \psi_j(t) \rangle = \gamma_j - \lim_{N\to\infty} \langle \sum_{i=0}^{N} \gamma_i \psi_i(t), \psi_j(t) \rangle = \gamma_j - \sum_{i=0}^{\infty} \gamma_i \langle \psi_i(t), \psi_j(t) \rangle = \gamma_j - \gamma_j = 0, \forall_j$. Hence, $y(t) - S \equiv 0 \Rightarrow y(t) \equiv S(= \sum_{j=0}^{\infty} \gamma_j \psi_j(t))$ i.e., $\sum_{j=0}^{\infty} \gamma_j \psi_j(t)$ converges to $y(t)$, **(exact solution)** $\forall t$.

5. Illustrative examples

To employ the presented method and its efficiency for solving the general Eq. (1) we consider this equation for different values of a(t), b(t) and g(t), and derive respective analytical solutions. The computations for these examples were performed in Maple 14.

Example 1. Consider Eq. (1) with:

$$w_p = 2, \quad a(t) = \cos(t), \quad b(t) = \sin\left(\frac{t}{2}\right),$$
$$g(t) = \cos(t) - t\sin(t) + \cos(t)\,(t\sin(t) + \cos(t)) - \sin\left(\frac{t}{2}\right)\left(\frac{2}{9}\sin(3t) - \frac{t}{6}\cos(3t) + \frac{t}{2}\cos(t)\right.$$

and α = 1, β = 0. The exact solution of this problem is y(t) = t sin(t) + cos(t)(see (Dehghan et al., 2008)). The numerical solution for Example (1) is obtained by the proposed method with M = 3 and k = 2 (8-terms). Table 1 represents the numerical results of this example. In (Maleknejad et al., 2013), for M = N = 2 (15-terms), the corresponding **error** is about 10^{-9}, for M = 4 and N = 5 (54-terms), the **error** is about 10^{-14}.

Example 2. In this example, we consider Eq. (1) with:

$$w_p = 1, \quad a(t) = -\sin(t), \quad b(t) = \sin(t),$$

Table 1. Numerical results for Example (1)

t	Absolute Error
0	9.7958 × 10-11
0.1	4.0445 × 10-13
0.2	3.6216 × 10-13
0.3	3.7054 × 10-13
0.4	3.9322 × 10-13
0.5	4.0165 × 10-13
0.6	9.9463 × 10-11
0.7	1.7852 × 10-12
0.8	1.7513 × 10-12
0.9	1.7689 × 10-12

$$g(t) = \frac{1}{9}e^{-\frac{t}{3}} - \sin(t)(e^{-\frac{t}{3}} + t) - \sin(t)\left(-\frac{3}{10}\cos(t)\,e^{-\frac{t}{3}} + \frac{9}{10}e^{-\frac{t}{3}}\sin(t) + \cos(t) + t\sin(t) - \frac{7}{10}\right)$$

and $\alpha = 1$, $\beta = \frac{2}{3}$. $y(t) = e^{-\frac{t}{3}} + t$ is the exact solution of this equation (see (Dehghan et al., 2008)).We solve this example with M = 3 and k = 2 (8-terms), and compare with the method in (Maleknejad et al., 2013),(6-terms, Table 2). Table 3 indicates the numerical results of this example with M = 5 and k = 2, (16-terms). In Maleknejad et al (2013), for M = N = 2 (15-terms), the **error** is about 10^{-7}, for M = 4 and N = 5 (54-terms), the **error** is about 10^{-11}.

Example 3. Finally, we consider Eq. (1) with:

$$w_p = 3, \quad a(t) = 1, \quad b(t) = \sin(t) + \cos(t),$$
$$g(t) = -t^3 + t^2 - 11t + 4 - (\sin(t) + \cos(t))$$
$$(-\frac{t^3}{3}\sin(3t) - \frac{t^3}{3}\cos(3t) + \frac{13}{27}\cos(3t) + \frac{13}{9}t\sin(3t) + \frac{t^2}{3}\sin(3t) + \frac{16}{27}\sin(3t) + \frac{2}{9}t\cos(3t) + \frac{13}{27})$$

and α = 2, β = −5. $y(t) = -t^3 + t^2 - 5t + 2$ is the exact solution of this equation (see (Dehghan et al., 2008)). The absolute **error** for M = 6 and k = 3 is listed in Table 4.

Table 2. Numerical results for Example (2)

t	AE our method	AE method of (Maleknejad et al., 2013)
0	1.2632×10^{-11}	4×10^{-4}
0.2	5.4356×10^{-11}	1×10^{-5}
0.4	9.1043×10^{-12}	1×10^{-5}
0.6	2.2734×10^{-11}	2×10^{-5}
0.8	3.0997×10^{-11}	2×10^{-5}

Table 3. Numerical results for Example (2)

t	Absolute Error
0	3.9293 × 10-14
0.1	4.0016 × 10-14
0.2	2.5215 × 10-13
0.3	2.7045 × 10-13
0.4	4.0072 × 10-14
0.5	4.1051 × 10-14
0.6	2.7134 × 10-13
0.7	3.8752 × 10-14
0.8	3.1597 × 10-13
0.9	3.0076 × 10-13

Table 4. Numerical results for Example (3)

t	Absolute Error
0	3.7958 × 10-14
0.1	4.8835 × 10-15
0.2	3.1626 × 10-14
0.3	6.0054 × 10-15
0.4	2.9029 × 10-14
0.5	4.7865 × 10-15
0.6	3.1004 × 10-14
0.7	5.5278 × 10-15
0.8	5.7975 × 10-15
0.9	6.1176 × 10-15

6. Conclusions

The aim of the present work is to propose an efficient method for solving an integro-differential equation which describes the charged particle motion for certain configurations of oscillating magnetic fields. The Chebyshev wavelets and collocation points have been applied for solving the problem by reducing the given integro-differential equation into a system of algebraic equations. The method is computationally attractive and applications are demonstrated through several illustrative examples.

Acknowledgments

We thank the referees for several detailed remarks that led to a substantial improvement in the content as well as the presentation of the paper. The authors would also like to thank Shahrekod University for the financial support through a research grant.

References

Bakar, A. H. A., Ali, M. S., Tan, C., Mokhlis, H., Arof, H., & Illias, H. A. (2014). High impedance fault location in 11 kV underground distribution systems using wavelet transforms. *International Journal of Electrical Power and Energy Systems*, 55(1), 723–730.

Bojeldain, A. A. (1991). On the numerical solving of nonlinear Volterra integro-differential equations. *Annales Universitatis Scientarium Budapestinensis de Rolando Eötvös Nominatae Sectio Computatorica*, 11, 105–125.

Canuto, C., Hussaini, M. Y., Quarteroni, A., & Zang, T. A. (1988). *Spectral Methods on Fluid Dynamics*. Springer Verlag.

Dehghan, M., & Shakeri, F. (2008). Solution of an integro-differential equation arising in oscillating magnetic fields using He's Homotopy Perturbation Method. *Progress in Electromagnetics Research*, 78, 361–376.

Ghasemi, M., Babolian, E., & Tavassoli Kajani, M. (2007). Numerical solution of linear Fredholm integral equations using sine-cosine wavelets. *International Journal of Computer Mathematics*, 84(7), 979–987.

Ghasemi, M., & Tavassoli Kajani, M. (2011). Numerical solution of time-varying delay systems by Chebyshev wavelets. *Applied Mathematical Modeling*, 35(11), 5235–5244.

Heydari, M., Avazzadeh, Z., & Loghmani, G. B. (2012). Chebyshev cardinal functions for solving volterra-fredholm integro-differential equations using operational matrices. *Iranian Journal of Science & Technology*, Transaction A, 34, 13–24.

Jiao, X., Ding, K., & He, G. (2014). An algorithm for improving the coefficient accuracy of wavelet packet analysis. *Measurement*, 47(1), 207–220.

Lashab, M., Benabdelaziz, F., & Zebiri C.E. (2007). Analysis of electromagnetics scattering from reflector and cylindrical antennas using wavelet-based moment method. *Progress in Electromagnetics Research*, doi: 10.2528/PIER07071401.

Lashab, M., Zebiri, C. E., & Benabdelaziz, F. (2008). Wavelet-based moment method and physical optics use on large reflector antennas. *Progress in Electromagnetics Research*, doi: 10.2528/PIERM08042902.

Machado, J. M., & Tsuchida, M. (2002). Solutions for a class of integro-differential equations with time periodic coefficients. *Applied Mathematics E-Notes*, 2, 66–71.

Maleknejad, K., Hadizadeh, M., & Attary, M. (2013). On the approximate solution of integro-differential equations arising in oscillating magnetic fields. *Application of Mathematics*, 58(5), 595–607.

Perez-Munoz, T., Velasco-Hernandez, J., & Hernandez-Martinez, E. (2013). Wavelet transform analysis for lithological characteristics identification in siliciclastic oil fields. *Journal of Applied Geophysics*, 98, 298–308.

Razzaghi, M., Oppenheimer, S., & Ahmad, F. (2002) A Legendre wavelet method for the radiative transfer equation in remote sensing. *Journal of Electromagnetic Waves and Applications*, 16(12) 1681–1693.

Razzaghi, M., & Yousefi, S. A. (2002). Sine-cosine wavelets operational matrix of integration and its applications in the calculus of variations. *International Journal of Systems Science*, 33(10), 805–810.

Razzaghi, M., & Shamsi, M. (2004). Efficient numerical techniques for solving Pocklington's integral equation using multiwavelets. *Journal of Electromagnetic Waves and Applications*, 18(2), 247–264.

Tavassoli Kajani, M., Ghasemi, M., & Babolian, E. (2006). Numerical solution of linear integro-differential equation by using sine-cosine wavelets. *Applied Mathematics and Computation*, 180(2), 569–574.

Tavassoli Kajani, M., Hadi Vencheh, A., & Ghasemi, M. (2009). The Chebyshev wavelets operational matrix of integration and product operation matrix. *International Journal of Computer Mathematics*, 86(7), 1118–1125.

Tavassoli Kajani, M., & Ghasemi, M. (2009). Chebyshev wavelets direct method for variational problems. *Proceedings of the 5 Asian Mathematical Conference*, Malaysia.

Tretiakov, Y., & Pan, G.W. (2004a). Malvar wavelet based Pocklington equation solutions to thinwire antennas and scatterers. *Progress in Electromagnetics Research*, doi: 10.2528/PIER03111701

Tretiakov, Y., & Pan, G.W. (2004b). Coifman wavelets in electromagnetic wave scattering by a groove in conducting plane. *Progress in Electromagnetics Research*, doi: 10.2528/PIER03091101.

Yan, R., Gao, R. X., & Chen, X. (2014). Wavelets for fault diagnosis of rotary machines: Areview with applications. *Signal Processing*, 96(1), 1–15.