Iranian Journal of Science & Technology, Transaction A, Vol. 33, No. A3
Printed in the Islamic Republic of Iran, 2009

FINSLER METRICS WITH SPECIAL LANDSBERG CURVATURE*

A. TAYEBI[1**] AND E. PEYGHAN[2]

[1]Department of Mathematics and Computer Science, Qom University, Qom, I. R. of Iran
Email: akbar.tayebi@gmail.com
[2]Department of Mathematics and Computer Science, Arak University, Arak, I. R. of Iran
Email: epeyghan@gmail.com

Abstract – In this paper, we study a class of Finsler metrics which contains the class of P-reducible and general relatively isotropic Landsberg metrics, as special cases. We prove that on a compact Finsler manifold, this class of metrics is nothing other than Randers metrics. Finally, we study this class of Finsler metrics with scalar flag curvature and find a condition under which these metrics reduce to Randers metric.

Keywords – Randers metric, flag curvature, Landsberg metric, P-reducible

1. INTRODUCTION

In Finsler geometry, there are several important non-Riemannian quantities. Let (M, F) be a Finsler manifold. The second derivatives of $\frac{1}{2}F_x^2$ at $y \in T_xM_0$ is an inner product g_y on T_xM. The third order derivatives of $\frac{1}{2}F_x^2$ at $y \in T_xM_0$ is a symmetric trilinear form C_y on T_xM. We call g_y and C_y the fundamental form and the Cartan torsion, respectively. The rate of change of the Cartan torsion along geodesics is the Landsberg curvature L_y on T_xM for any $y \in T_xM_0$. Set $\mathrm{J}_y := \sum_{i=1}^{n} L_y(e_i, e_i, .)$, where $\{e_i\}$ is an orthonormal basis for (T_xM, g_y). J_y is called the mean Landsberg curvature. F is said to be Landsbergian if $\mathrm{L} = 0$, and weakly Landsbergian if $\mathrm{J} = 0$ [1, 2].

Various interesting special forms of Cartan and Landsberg tensors have been obtained by some Finslerians. The Finsler spaces having such special forms have been called C-reducible, P-reducible, general relatively isotropic Landsberg, and etc. In [3], Matsumoto introduced the notion of C-reducible Finsler metrics and proved that any Randers metric is C-reducible. Later on, Matsumoto-Hōjō proves that the converse is true too [4]. A Randers metric $F = \alpha + \beta$ is just a Riemannian metric α perturbated by a one form β. Randers metrics have important applications in both mathematics and physics [5]. As a generalization of C-reducible metrics, Matsumoto-Shimada introduced the notion of P-reducible metrics [6]. This class of Finsler metrics has some interesting physical means and contains Randers metrics as a special case.

In [7], Prasad introduced a new class of Finsler spaces which contains the notion of P-reducible and general relatively isotropic Landsberg spaces, as special cases. Let us put

$$L_{ijk} = \lambda C_{ijk} + a_i h_{jk} + a_j h_{ki} + a_k h_{ij}, \tag{1}$$

where $\lambda = \lambda(x, y)$ and $a_i = a_i(x, y)$ are scalar functions on TM and $h_{ij} = g_{ij} - F^{-2} y_i y_j$ is the angular metric. λ and a_i are the homogeneous function of degree 1 and degree 0 with respect to y,

---

*Received by the editor June 22, 2009 and in final revised form February 20, 2010
**Corresponding author

respectively. By definition, we have $a_i\, y^i = 0$. Therefore, the study of this class of Finsler spaces will enhance our understanding of the geometric meaning of Randers metrics. If $a_i = 0$, then F is reduce to a general isotropic Landsberg metric and if $\lambda = 0$, then F is a P-reducible metric.

Let F be a Landsberg metric satisfied in (1). Then F is a C-reducible metric. In a 1974 paper [3], Matsumoto showed that $F = \alpha + \beta$ is a Landsberg metric if and only if β is parallel. In a 1977 paper [8], M. Hashiguchi and I. Ichijyo showed that for a Randers metric $F = \alpha + \beta$, if β is parallel, then F is a Berwald metric. Then every Landsberg metric satisfyed in (1) is Berwaldian.

In this paper, we prove that on a compact Finsler manifold, this class of metrics reduces to the class of Randers metrics. More precisely, we prove the following.

Theorem 1. Let (M, F) be a compact Finsler manifold with dimension $n \geq 3$. Suppose that F satisfy in the equation (1). Then F is a Randers metric.

Then we study this class of Finsler metrics with scalar flag curvature and find a condition under which these metrics reduce to a Randers metric. More precisely, we prove the following.

Theorem 2. Let (M, F) be a Finsler manifold of scalar flag curvature K with dimension $n \geq 3$. Suppose that F satisfy in the equation (1) with $\lambda_{|i}\, y^i + \lambda^2 + K \neq 0$. Then F is a Randers metric.
There are many connections in Finsler geometry [9-11]. Throughout this paper, we set the Berwald connection on Finsler manifolds. The h- and v- covariant derivatives of a Finsler tensor field are denoted by " | " and ", " respectively.

2. PRELIMINARIES

Let M be an n-dimensional C^∞ manifold. Denote by T_xM the tangent space at $x \in M$, by $TM = \bigcup_{x \in M} T_x M$ the tangent bundle of M, and by $TM_0 := TM \setminus \{0\}$ the slit tangent bundle of M. A Finsler metric on M is a function $F : TM \to [0,\infty)$ which has the following properties: (i) F is C^∞ on TM_0; (ii) F is positively 1-homogeneous on the fibers of tangent bundle TM, and (iii) for each $y \in T_xM$, the following quadratic form g_y on T_xM is positive definite,

$$\mathrm{g}_y(u,v) := \frac{1}{2}\,[F^2(y + su + tv)]_{s,t=0}\ , \qquad u,v \in T_xM\ .$$

Let (M, F) be a Finsler manifold of dimension n. Fix a local frame $\{b_i\}$ for TM. The Finsler metric $F = F(y^i\, b_i)$ is a function of (x^i, y^i). Let

$$C_{ijk}(x,y) := \frac{1}{4}[F^2]_{y^i y^j y^k}(x,y).$$

For a non-zero vector $y = y^i b_i \in T_xM$, the Cartan torsion C_y on T_xM is a trilinear symmetric form on T_xM defined by $\mathrm{C}_y(b_i, b_j, b_k) := C_{ijk}(x,y)$. The mean Cartan torsion I_y is a linear form on $y = y^i b_i \in T_xM$ defined by

$$\mathrm{I}_y(b_i) = I_i(x,y) := g^{jk}(x,y)\ C_{ijk}(x,y).$$

The spray of F is a vector field on TM_0. In a standard local coordinate system (x^i, y^i) in TM, the spray is given by

$$\mathrm{G} = y^i \frac{\partial}{\partial x^i} - 2G^i(x,y)\frac{\partial}{\partial y^i}$$

where $G^i(y):=\frac{1}{4}g^{il}(x,y)\{[F^2]_{x^k y^l}y^k-[F]^2_{x^l}(y)\}$. A Finsler metric F is called a Berwald metric if $G^i(x,y):=\frac{1}{2}\Gamma^i{}_{jk}(x)y^j y^k$ are quadratic in $y\in T_xM$. It is known that every Berwald metric has the same geodesics as a Riemannian metric [12]. The local structures of Berwald metrics have been completely determined by Z. I. Szabo [13]. Thus Berwald metrics can be identified with Riemannian metrics at geodesic level.

Let $c(t)$ be a C^∞ curve and $U(t)=U^i(t)\frac{\partial}{\partial x^i}|_{c(t)}$ be a vector field along c. Define the covariant derivative of $U(t)$ along c by

$$D_{\dot c}U(t):=\{\frac{dU^i}{dt}(t)+U^j(t)\frac{\partial G^i}{\partial y^j}(c(t),\dot c(t))\}\frac{\partial}{\partial x^i}|_{c(t)}$$

$U(t)$ is said to be linearly parallel if $D_{\dot c}U(t)=0$.

For a vector $y\in T_xM$, define

$$\mathrm{L}_y(u,v,w):=\frac{d}{dt}[\mathrm{C}_{\dot\sigma(t)}(U(t),V(t),W(t))]|_{t=0},$$

$$\mathrm{J}_y(u):=\frac{d}{dt}[\mathrm{I}_{\dot\sigma(t)}(U(t))]|_{t=0},$$

where $\sigma(t)$ is the geodesic with $\sigma(0)=x$, $\dot\sigma(0)=y$ and $\mathrm{U(t), V(t), W(t)}$ are linearly parallel vector fields along σ with $\mathrm{U(0)=u,\ V(0)=v,\ W(0)=w}$. We call L_y the Landsberg curvature. The Landsberg curvature measures the rate of change of the Cartan torsion along the geodesics. Let $L_{ijk}(x,y):=\mathrm{L}_y(b_i,b_j,b_k)$ and $J_i(x,y):=\mathrm{J}_y(b_i)$. We have that $J_i(x,y)=g^{jk}(x,y)L_{ijk}(x,y)$. Thus we call J_y the mean Landsberg curvature [1].

L/**C** is regarded as the relative rate of change of **C** along the geodesics. A Finsler metric F on a manifold *M* is said to be a general relatively isotropic Landsberg metric if $\mathrm{L}=\mu\mathrm{C}$, where μ is a positively 1-homogeneous scalar function on TM_0 [14]. The generalized Funk metrics on the unit ball $B^n\subset R^n$ satisfy **L**+cF**C**=0 for some constant $c\neq 0$ [15]. To the same way, **J**/**I** is regarded as the relative rate of change of **I** along the geodesics and F is said to be a general relatively isotropic mean Landsberg metric if $\mathrm{J}=\mu\mathrm{I}$ [16].

For a non-zero vector $y\in T_xM$, the tensor **T** induces a multi-linear form $T_y(u,\cdots,v):=T_{i\cdots k}(x,y)\,u^i\cdots w^k$ on T_xM. Let $\sigma(t)$ denote the geodesic with $\dot\sigma(0)=y$. We have

$$\frac{d}{dt}[\mathrm{T}_{\dot\sigma(t)}(U(t),\cdots,W(t))]=T_{i\cdots k|m}(\sigma(t),\dot\sigma(t))\dot\sigma^m(t)\ U^i(t)\cdots W^k(t)$$

where $U(t)=U^i(t)\frac{\partial}{\partial x^i}|_{c(t)},\cdots,W(t)=W^k(t)\frac{\partial}{\partial x^k}|_{c(t)}$ are linearly parallel vector fields along σ. Thus the **L**-curvature $\mathrm{L}=L_{ijk}\,w^i\otimes w^j\otimes w^k$ and the **J**-curvature $\mathrm{J}=J_i\,w^i$ are given by

$$L_{ijk}=C_{ijk|m}\,y^m,\qquad J_i=I_{i|m}y^m. \tag{2}$$

Let

$$M_{ijk}:=C_{ijk}-\frac{1}{n+1}\{I_i\,h_{jk}+I_j\,h_{ki}+I_k\,h_{ij}\}.$$

We obtain a symmetric trilinear form M_y on T_xM defined by $\mathrm{M}_y(b_i,b_j,b_k):=M_{ijk}(x,y)$. This quantity is introduced by M. Matsumoto [17]. Thus we call M_y the Matsumoto torsion. Matsumoto proves that every Randers metric satisfies that $\mathrm{M}_y = 0$. Later on, Matsumoto-Hōjō proves that the converse is true too.

Lemma 1. ([17][4]) A Finsler metric F on a manifold of dimension $n \geq 3$ is a Randers metric if and only if $\mathrm{M}_y = 0$, $\forall\, y \in TM_0$.

Finsler metrics in this paper are always assumed to be regular in all directions. If this regularity is not imposed, Matsumoto-Hōjō's theorem says that F has vanishing Matsumoto torsion if and only if $F = \alpha + \beta$ or $F = \frac{\alpha^2}{\beta}$, where α is a Riemannian metric and β is a 1-form on *M*.

Define $\overline{\mathrm{M}}_y : T_xM \otimes T_xM \otimes T_xM \to R$ by $\overline{\mathrm{M}}_y(u,v,w) := \overline{M}_{ijk}(y)\, u^i v^j w^k$ where

$$\overline{M}_{ijk} := L_{ijk} - \frac{1}{n+1}\{J_i\, h_{jk} + J_j\, h_{ki} + J_k\, h_{ij}\}.$$

A Finsler metric F is said to be P-reducible if $\overline{\mathrm{M}}_y = 0$. The notion of P-reducibility was given by Matsumoto-Shimada [6]. It is obvious that every C-reducible metric s a P-reducible metric.

The Riemann curvature $\mathbf{K}_y = K^i_k dx^k \otimes \frac{\partial}{\partial x^i}|_x : T_xM \to T_xM$ is a family of linear maps on tangent spaces, defined by

$$K^i_k = 2\frac{\partial G^i}{\partial x^k} - y^j \frac{\partial^2 G^i}{\partial x^j \partial y^k} + 2G^j \frac{\partial^2 G^i}{\partial y^j \partial y^k} - \frac{\partial G^i}{\partial y^j}\frac{\partial G^j}{\partial y^k}.$$

For a flag $P = \mathrm{span}\{y,u\} \subset T_xM$ with flagpole *y*, the flag curvature $\mathbf{K} = \mathbf{K}(P,y)$ is defined by

$$\mathbf{K}(P,y) := \frac{\mathbf{g}_y(u, \mathbf{K}_y(u))}{\mathbf{g}_y(y,y)\mathbf{g}_y(u,u) - \mathbf{g}_y(y,u)^2},$$

where $\mathbf{g}_y = g_{ij}(x,y)dx^i \otimes dx^j$. When *F* is Riemannian, $\mathbf{K} = \mathbf{K}(P)$ is independent of $y \in P$, which is just the sectional curvature of *P* in Riemannian geometry. We say that a Finsler metric *F* is of scalar curvature if for any $y \in T_xM$, the flag curvature $K = K(x,y)$ is a scalar function on the slit tangent bundle TM_0. If **K**=*constant*, then *F* is said to be of constant flag curvature.

3. PROOF OF THEOREM 1

In this section, we are going to prove a generalization of Theorem 1. First, we define the norm of the Matsumoto torsion at $x \in M$ by

$$\|\mathrm{M}\|_x := \sup_{y,u,v,w \in T_xM_0} \frac{F(y)\,|\mathrm{M}_y(u,v,w)\,|}{\sqrt{g_y(u,u)\; g_y(v,v) g_y(w,w)}}.$$

Theorem 3. Let $(M,\, F)$ be a complete Finsler manifold satisfied in equation (1) with dimension $n \geq 3$. Suppose that F has bounded Matsumoto torsion. Then F is a Randers metric.

Proof: We will first prove that the Matsumoto torsion vanishes. To prove this, we assume that the Matsumoto torsion $\mathrm{M}_y(u,u,u) = M_{ijk}(x,y)u^i u^j u^k \neq 0$ for some $y,\, u \in T_xM_0$ with $\mathrm{F(x,y)} = 1$. Let $\sigma(t)$ be the unit speed geodesic with $\sigma(0) = x$ and $\dot{\sigma}(0) = y$. Let $U(t)$ denote the linear parallel vector

field along σ, that is, $D_{\dot\sigma}U(t)=0$. From the above equation, we see that a linearly parallel vector field $U(t)$ along σ linearly depends on its initial value $U(t_0)$ at a point $\sigma(t_0)$.
Let

$$M(t):=\mathrm{M}_\sigma(U(t),U(t),U(t))=M_{ijk}(\sigma(t),\dot\sigma(t))U^i(t)U^j(t)U^k(t).$$

We have

$$M'(t)=M_{ijk|p}\ \ (\sigma(t),\dot\sigma(t))\ \ \dot\sigma^p(t)\ \ U^i(t)U^j(t)U^k(t)$$

Now we assume that *F* is satisfied in the equation (1):

$$L_{ijk}=\lambda\, C_{ijk}+a_i\, h_{jk}+a_j\, h_{ki}+a_k\, h_{ij}, \tag{3}$$

Contacting (3) with g^{ij} and using the relations $g^{ij}h_{ij}=n-1$ and $g^{ij}(a_i\, h_{jk})=g^{ij}(a_j\, h_{ik})=a_k$ implies that

$$J_k=\lambda\, I_k+(n+1)\,a_k. \tag{4}$$

Then

$$a_i=\frac{1}{n+1}J_i-\frac{\lambda}{n+1}\,I_i. \tag{5}$$

Putting (5) in (3) yields

$$\begin{aligned}L_{ijk}=\lambda\, C_{ijk}&+\frac{1}{n+1}\{J_i\, h_{jk}+J_j\, h_{ki}+J_k\, h_{ij}\}\\&-\frac{\lambda}{n+1}\{I_i\, h_{jk}+I_j\, h_{ki}+I_k\, h_{ij}\}.\end{aligned} \tag{6}$$

By simplifying (6), we get

$$L_{ijk}-\frac{1}{n+1}(J_i\, h_{jk}+J_j\, h_{ki}+J_k\, h_{ij})=\lambda\,\{C_{ijk}-\frac{\lambda}{n+1}(I_i\, h_{jk}+I_j\, h_{ki}+I_k\, h_{ij})\}. \tag{7}$$

The equation (7) is equivalent to

$$M_{ijk|s}\, y^s=\lambda(x,y)M_{ijk}. \tag{8}$$

It follows from (8) that

$$M'(t)=\lambda(t)\ M(t). \tag{9}$$

Take an arbitrary unit vector $y\in T_xM$ and an arbitrary vector $v\in T_xM$. Let $\mathrm{c(t)}$ be the geodesic with $\dot c(0)=y$ and $V(t)$ the parallel vector field along *c* with $V(0)=v$. From equation (9), we have

$$M(t)=ce^{\lambda t}. \tag{10}$$

Since *M* is complete and $\|M\|<\infty$, by letting $t\to+\infty$ or $t\to-\infty$, we have *c=0*. Thus the Matsumoto torsion vanishes. By Lemma 1, *F* must be a Randers metric.
By the relation (7), we get the following corollaries.

Corollary 1. Let F be a Finsler metric satisfied in the equation (1) with dimension $n \geq 3$ and $\lambda \neq 0$. Then F is C-reducible if and only if F is P-reducible.
By a simple calculation on the equation (7), we have the following.

Corollary 2. Let F be a Finsler metric satisfied in the equation (1) with dimension $n \geq 3$ and $\lambda \neq 0$. Then the following are equivalent:
(a) F has a general relatively isotropic Landsberg curvature;
(b) F has a general relatively isotropic mean Landsberg curvature.

4. PROOF OF THEOREM 2

Lemma 1. ([18, 19]) Landsberg curvature and Riemann curvature are related by the following equation

$$L_{ijk|m}\, y^m + C_{ijm}\, R^m{}_k = -\frac{1}{3}\, g_{im}\, R^m{}_{k,j} - \frac{1}{3}\, g_{jm}\, R^m{}_{k,i} - \frac{1}{6}\, g_{im}\, R^m{}_{j,k} - \frac{1}{6}\, g_{jm}\, R^m{}_{i,k}. \tag{11}$$

Contracting (11) with g^{ij} gives

$$J_{k|m}\, y^m + I_m\, R^m{}_k = -\frac{1}{3}\{2K^m{}_{k,m} + K^m{}_{m,k}\}. \tag{12}$$

Proof of Theorem 2. We will first prove that the Matsumoto torsion vanishes. To prove this, we assume that the Matsumoto torsion $\mathrm{M}_y(u,u,u) = M_{ijk}(x,y)u^i u^j u^k \neq 0$ for some $y, u \in T_x M_0$ with $\mathrm{F(x,y)} = 1$. Let $\sigma(t)$ be the unit speed geodesic with $\sigma(0) = x$ and $\dot\sigma(0) = y$. Let $U(t)$ denote the linear parallel vector field along σ, that is, $D_{\dot\sigma}U(t) = 0$. From the above equation, we see that a linearly parallel vector field $U(t)$ along σ linearly depends on its initial value $U(t_0)$ at a point $\sigma(t_0)$.
Let

$$M(t) := \mathrm{M}_\sigma(U(t),U(t),U(t)) = M_{ijk}(\sigma(t),\dot\sigma(t))U^i(t)U^j(t)U^k(t).$$

We have

$$M''(t) = M_{ijk|p|q}\ (\sigma(t),\dot\sigma(t))\ \dot\sigma^p(t)\dot\sigma^q(t)\ \ U^i(t)U^j(t)U^k(t)$$

Now we assume that *F* is of scalar curvature with flag curvature $\mathrm{K} = \mathrm{K(x,y)}$. This is equivalent to the following identity:

$$R^i{}_k = KF^2 h^i_k, \tag{13}$$

where $h^i{}_k := g^{ij}\, h_{jk}$. Differentiating (13) yields

$$R^i{}_{k,l} = K_{,l}\, F^2 h^i_k + K\{2g_{lp}\, y^p \delta^i_k - g_{kp}\, y^p \delta^i_k - g_{kl} y^i\} \tag{14}$$

By (11), (12) and (14), we obtain

$$L_{ijk|m}\, y^m = -\frac{1}{3}F^2\{K_{,i}h_{jk} + K_{,j}h_{ik} + K_{,k}h_{ji} + 3KC_{ijk}\} \tag{15}$$

And

$$J_{k|m}\, y^m = -\frac{1}{3} F^2 \{(n+1)K_{,k} + 3KI_k\} \tag{16}$$

By (2), we have

$$C_{ijk|p|q} y^p y^q = L_{ijk|m}\, y^m, \qquad I_{k|p|q} y^p y^q = J_{k|m}\, y^m.$$

$$M_{ijk|p|q}\, y^p\, y^q := L_{ijk|m}\, y^m - \frac{1}{n+1}\{J_{i|m}\, y^m\, h_{jk} + J_{j|m}\, y^m\, h_{ki} + J_{k|m}\, y^m\, h_{ij}\}. \tag{17}$$

Plugging (15) and (16) into (17) yields

$$M_{ijk|p|q}\, y^p\, y^q + K F^2 M_{ijk} = 0. \tag{18}$$

It follows from (18) that

$$M''(t) + K(t)\, M(t) = 0 \tag{19}$$

By (9) we have

$$M''(t) = \lambda'\, M(t) + \lambda\, M'(t) = (\lambda' + \lambda^2) M. \tag{20}$$

By (20) and (19) we get

$$(\lambda' + \lambda^2 + K) M = 0 \tag{21}$$

By assumption $\lambda' + \lambda^2 + K \neq 0$, then $M = 0$. This completes the proof.

REFERENCES

1. Shen, Z. (2001). Lectures on Finsler Geometry. *World Scientific Publishers*.
2. Shen, Z. (2000). Differential Geometry of Spray and Finsler Spaces. *Kluwer Academic*.
3. Matsumoto, M. (1974). On Finsler spaces with Randers metric and special forms of important tensors. *J. Math. Kyoto Univ*, 477-498.
4. Matsumoto, M. & Hōjō, S. (1978). A conclusive theorem for C-reducible Finsler spaces. *Tensor. N. S*, 225-230.
5. Randers, G. (1941). On an asymmetric metric in the four-space of general relativity. *Phys. Rev*, 195-199.
6. Matsumoto, M. & Shimada, H. (1977). On Finsler spaces with the curvature tensors P_{hijk} and R_{hijk} satisfying special conditions. *Rep. Math. Phys*, 77-87.
7. Prasad, B. N. (1980). Finsler spaces with the torsion tensor P_{ijk} of a special form, *Indian. J. Pur. Appl. Math*, 1572-1579.
8. Hashiguchi, M. & Ichijyō, Y. (1975). On some special (α, β)-metrics. *Rep. Fac. Sci., Kagoshima Univ*, 39-46.
9. Bidabad, B. & Tayebi, A. (2009). Properties of generalized Berwald connections, Bull. *Iran. Math. Society. 1*(35), 237-254.
10. Bidabad, B. & Tayebi, A. (2007). A classification of some Finsler connections. *Publ. Math. Debrecen. 71*, 253-260.
11. Tayebi, A., Azizpour, E. & Esrafilian, E. (2008). On a family of connections in Finsler geometry. *Publ. Math. Debrecen*, *72*, 1-15.
12. Szabó, Z. I. (1977). Ein Finslerscher Raum ist gerade dann von skalarer Kr¨ummung, wenn seine Weyl sche Projektivkr¨umm¨ung verschwindet. *Acta Sci. Math*, 163-168.
13. Szabó, Z. I. (1981). Positive definite Berwald spaces (Structure theorems on Berwald spaces). *Tensor, N. S*, 25-39.

14. Najafi, B., Tayebi, A. & Rezaei, M. M. (2005). General Relatively Isotropic L-curvature Finsler manifolds, *IJST, Trans. A*, 357-366.
15. Chen, X. & Shen, Z. (2003). Randers metrics with special curvature properties. *Osaka J. of Math*, 87-101.
16. Najafi, B., Tayebi, A. & Rezaei, M. M. (2005). General Relatively Isotropic Mean Landsberg Finsler manifolds, *IJST, Trans. A*, 497-505.
17. Matsumoto, M. (1972). On C-reducible Finsler spaces. *Tensor, N. S*, 29-37.
18. Mo, X. (1999). The flag curvature tensor on a closed Finsler space. *Results in Math*. 149-159.
19. Tayebi, A. & Peyghan, E. (2010). On Ricci tensors of Randers metrics, *Journal of Geometry and Physics, 60*, 1665-1670.