ESTIMATION OF GALAXY AGE BY MODIFIED MEYER AND SHRAMM NUCLEOSYNTHESIS MODEL

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Abstract – The independent model used by Meyer and Schramm (here after M&S) for determining the age of elements (chemical evolution) is extended. Using new and extended definitions and including parameters ignored by M&S, the galactic evolution equations are solved to determine the total duration of nucleosynthesis, T. To be able to calculate the $T/t_{\text{e}}$ (mean age time for formation of elements to total duration of nucleosynthesis ratio), one event age, $\Delta_{\text{meq}}$, and $r(i, j)$ are determined using approximation relating $r'(i, j)$ to the former value $r(i, j)$. For Th/U pair, constraint on $T/t_{\text{e}}$ becomes $61.0 \leq T/t_{\text{e}} \leq 33.0$ and the age of the galaxy is obtained as $12.5 \leq T_{\text{Gal}} \leq 23.0$, reducing the variation considerably compared to M&S formalism ($1.28 \leq T_{\text{Gal}} \leq 28.1$). The range of constraints for the age of the universe is shorter than the M&S and our modifications on the model have reduced the interval for galaxy age 83% and close to being concordant with other available methods.

Keywords – Nucleosynthesis, cosmochronology, age of universe, independent model, galaxy evolution, chronometric pairs, abundances

1. INTRODUCTION

The evolution of the universe, from our perspective, can be divided into four stages: 1. Big bang primordial nucleosynthesis leading to the formation of neutral atoms (p.n.~10^6 yr). 2. Condensation of galaxies and first generation stars (time interval=δ~1.5±0.5 Gyr). 3. Nucleosynthesis in stars and supernovae leading to the formation of the present chemical elements (time interval T=0 to T=T+Δ). Finally, condensation of the solar system from the debris of earlier stars (time interval $t_{\text{ss}}$=4.55 Gyr). Having $t_{\text{as}}$ from meteorite mass spectroscopy (comparison of abundance for daughter and mother radionuclides), and having $t=T+\Delta$ from nuclear calculation, and calculating $\delta$ from gravitational physics, the age for the universe can be approximated by $T_{\text{Gal}}=p.n+\delta+T+t_{\text{ss}}$, of which p.n. is certainly very small compared to other terms and can be neglected [1]. Nucleocosmochronology employs a knowledge of abundance and production ratio of radioactive nuclides and of the chemical evolution of the galaxy to obtain information about the time scale over which the solar system elements were synthesized. There are several methods relevant to model starting from Rutherford (1929). He established a model to determine the synthesis of Uranium isotopes. In 1957 Burbidge and et al., suggested the chemical evolution of the galaxy based on cosmochronology. In 1960 Fowler and Hoyle suggested an exponentially decaying synthesis model for chronometric pairs like $^{235}\text{U} / ^{238}\text{U}$ and $^{232}\text{Th} / ^{238}\text{U}$ consisting of uniform and sudden synthesis. In 1964 Clyton used $^{187}\text{Re} / ^{187}\text{Os}$ chronometric pair which holds great promise for accurate determination of the galaxy’s age. Despite the considerable amount of work done on nucleochronology, many uncertainties in nuclear and meteoritic data preclude accurate conclusions for the

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galaxy’s age. Furthermore, assumptions made for the model to galactic chemical evolution are also a source of error in age estimates for the galaxy. If a wrong model is chosen, many errors would emerge in age determination [2]. For this reason, in 1986 M & S turned to the independent cosmochronology model which was supposed by Schramm and Wasserburg in 1970 [3]. M&S obtained an upper limit on the galaxy’s age using an independent model. In this paper we revise the M&S procedure with some new definitions and using our extended model, it is shown that the calculated galaxy’s age range is reduced.

2. BASIC EQUATION AND GALAXY EVOLUTION FRAMEWORK

Suppose that in the interstellar medium of the galaxy, the abundance of materials going into the media are homogeneous, so the general equation governing the time evolution of the abundance $N_i$ of nuclide $i$ is,

$$\frac{dN_i(t)}{dt} = P_i \psi(t) - \lambda_i N_i(t) - \omega(t) N_i(t)$$

(1)

where $P_i$ is the number of nuclei produced per unit mass going into the stars (production rate) and $\psi(t)$ is the amount of mass going into stars per unit time (star formation rate) [2], this factor is known from the astronomical observation of UB and radio wave radiation [4]. $\lambda_i$ is the decay constant for nucleus $i$, $\omega(t)$ is a time dependent parameter representing the rate of movement of metals into and out of the interstellar medium for reasons other than radioactivity decay (e.g. loss into stellar remnants) [2, 5]. Mass loss rate $\omega(t)$ consists of at least two origins: first, the loss rate due to remnant formation which can be calculated in structure evolution models with various progenitor mass of main sequence stars, and second, the dynamical loss rate due to stellar and galactic wind which can be determined from the observed stellar metallicity distribution in a one-zone simple Galactic Chemical Evolution (GCE) model [4]. In a stellar structure, at the onset of the supernovae stage silicon burning or e-process (for equilibrium), rapid process and proton capture or the p-process also occur in supernovae and are responsible for heavy metal isotopes. Fe-group elements may also be synthesized by the e-process in Type I supernovae. Except for the production of $^7Li$ in the Big Bang, the lighter metals like Li, Be and B are not produced in any of the above processes. They are believed to be formed by the interactions of cosmic rays with interstellar gas and dust [6].

Equation (1) is a first order linear inhomogeneous differential equation. Solving this equation gives us abundance $N_i(T)$ of nuclide $i$ at time $T$ as a function of time $t$:

$$N_i(T) = e^{-\lambda_i T - \psi(T)} \int_0^T P_i \psi(t) e^{\lambda_i t + \nu(t)} dt$$

(2)

where $T$ is the time of the last event contributing to the formation of the elements going into the solar system. $\Delta$ is supposed to be the time interval between the last nucleosynthetic event and the solidification of the solid body in the solar system (the period of free decay for elements). We can have the abundance $N_i(T + \Delta)$ of nuclide $i$ at time $(T+\Delta)$ by using Taylor expansion [2]:

$$N_i(T + \Delta) = P_i \psi(T) \Delta + e^{\lambda_i \Delta - \nu(T)} e^{\lambda_i \Delta - \psi(T)} \int_0^T P_i \psi(t) e^{\lambda_i t + \nu(t)} dt$$

(3)

where $\nu(T)$ is the metal variation at the interstellar medium during the nucleosynthesis interval and is defined as:

$$\nu(T) = \int_0^T \omega(T) dt.$$  

(4)
The abundance $N_i(T + \Delta)$ in equation (3) is dependent upon the effective nucleosynthesis rate $\psi e^\nu$. M&S defined $\phi(t)$, the normalized effective nucleosynthesis rate[2, 5], as

$$\phi(t) = \frac{\psi e^\nu}{\langle \psi \rangle_T} \tag{5}$$

where $\langle \psi \rangle$, the average effective nucleosynthesis rate [2,4], is defined as:

$$\langle \psi \rangle \equiv \frac{1}{T} \int_0^T \psi e^\nu \, dt. \tag{6}$$

From equation (5) instead of $\psi e^\nu$ we can use $T \langle \psi \rangle \phi(t)$, therefore equation (3) becomes

$$N_i(T + \Delta) = P_i \psi(T) \Delta + e^{-\lambda_i \Delta - \alpha_i(T) \Delta} \int_0^T P_i e^{\lambda_i t} < \psi > \phi(t) \, dt. \tag{7}$$

Similar to M&S, we use Tinsley’s definition of the mean time for the formation of the elements [2, 7], therefore:

$$t_v \equiv \frac{\int_0^T \psi e^\nu t \, dt}{\int_0^T \psi e^\nu \, dt} = \frac{1}{T} \int_0^T t \psi(t) e^\nu(t) \, dt = \int_0^T t \phi(t) \, dt. \tag{8}$$

By multiplying and dividing the second term in equation (7) to $e^{-\lambda_i t_v}$, we have:

$$N_i(T + \Delta) = P_i \psi(T) \Delta + P_i T \langle \psi \rangle e^{-\lambda_i \Delta - \alpha_i(T) \Delta - \nu_i(T)} e^{-\lambda_i (t_v - t_v)} \int_0^T e^{\lambda_i (t - t_v)} \phi(t) \, dt. \tag{9}$$

Using a similar definition made by M&S for $\delta_i[2]$:

$$\delta_i = \sum_{n=2}^{\infty} \frac{\lambda_i}{n!} \mu_n \tag{10}$$

where $\mu_n$ is the nth moment of $\phi(t)$ about $t_v[2]$, therefore:

$$\mu_n = \int_0^T (t - t_v)^n \phi(t) dt. \tag{11}$$

Expanding equation (9) in moments of the normalized effective nucleosynthesis rate $\phi(t)$, we have:

$$N_i(T + \Delta) = P_i \psi(T) \Delta + P_i T \langle \psi \rangle e^{-\lambda_i \Delta - \alpha_i(T) \Delta - \nu_i(T)} (1 + \delta_i). \tag{12}$$

Substituting $\psi(t)$ in the first term of equation (12) by $T \langle \psi \rangle \phi(t) e^\nu$ from equation (5), we have:

$$N_i(T + \Delta) = P_i \Delta T \langle \psi \rangle \phi(t) e^\nu(t) + P_i T \langle \psi \rangle e^{-\lambda_i \Delta - \alpha_i(T) \Delta - \nu_i(T)} (1 + \delta_i). \tag{13}$$

Using equation (13) for two different nuclides of $i$ and $j$, their abundance ratio can be determined:

$$\frac{N_i(T + \Delta)}{N_j(T + \Delta)} = \frac{P_i \Delta T \langle \psi \rangle \phi(t) e^\nu(t) \Delta \phi_j(t) + (1 + \delta_i) e^{-\lambda_i \Delta - \alpha_i(T) \Delta}}{P_j \Delta T \langle \psi \rangle \phi(t) e^\nu(t) \Delta \phi_j(t) + (1 + \delta_j) e^{-\lambda_j \Delta - \alpha_j(T) \Delta}}. \tag{14}$$
Taking the natural logarithm from equation (14), and using two new definitions for \( t_v \) and \( T \) as,

\[
t_v = \frac{\ln T \langle \psi \rangle e^{-\psi_i}}{\lambda_i - \lambda_j}
\]

and

\[
T = \frac{\ln T \langle \psi \rangle e^{-\psi_i}}{\lambda_i - \lambda_j}
\]

we have

\[
T - t_v = \frac{\ln \left( \frac{P_i}{P_j} \right) \left[ \frac{N_i (T + \Delta)}{N_j (T + \Delta)} \right]}{\lambda_i - \lambda_j} - \frac{2}{3} \Delta + \frac{2\Delta (\omega_i - \omega_j)}{3(\lambda_i - \lambda_j)} + \frac{1}{3(\lambda_i - \lambda_j)} \ln \frac{\delta_i}{\delta_j} + \frac{1}{3(\lambda_i - \lambda_j)} \ln \frac{\phi_i}{\phi_j}.
\]

Defining one event age \( \Delta_{ij}^{max} \) [2, 5], as:

\[
\Delta_{ij}^{max} = \frac{\ln \left( \frac{P_i}{P_j} \right) \left[ \frac{N_i (T + \Delta)}{N_j (T + \Delta)} \right]}{\lambda_i - \lambda_j} \equiv \frac{\ln R (i, j)}{\lambda_i - \lambda_j}
\]

and for \( \omega_i = \omega_j \), equation (17) reduces to:

\[
T - t_v = \Delta_{ij}^{max} - \frac{2}{3} \Delta + \frac{1}{3(\lambda_i - \lambda_j)} \ln \frac{\phi_i(t)}{\phi_j(t)} + \frac{1}{3(\lambda_i - \lambda_j)} \ln \frac{\delta_i}{\delta_j}.
\]

Since \( \delta_i \) is proportional to \( \mu_n \lambda_i^m \) and \( \mu_n \) is essentially proportional to \( T^n \), \( \delta_i \) is proportional to \( \lambda_i^{m+1} T^n \). For long-lived chronometers (\( \lambda T \ll 1 \)), equation (19) reduces to:

\[
T - t_v = \Delta_{ij}^{max} - \frac{2}{3} \Delta + \frac{1}{3(\lambda_i - \lambda_j)} \ln \frac{\phi_i(t)}{\phi_j(t)} + \frac{1}{3(\lambda_i - \lambda_j)} \ln \frac{\delta_i}{\delta_j}
\]

and \( T \) can be written as:

\[
T = \left( 1 - \frac{t_v}{T} \right)^{-1} \left( \Delta_{ij}^{max} - \frac{2}{3} \Delta + \frac{1}{3(\lambda_i - \lambda_j)} \ln \frac{\phi_i(t)}{\phi_j(t)} \right).
\]

### 3. CONSTRAINTS ON MEAN LIFE-TIME FOR FORMATION OF ELEMENTS TO TOTAL DURATION OF NUCLEOSYNTHESIS RATIO

In this section, again we use the M&S procedure to obtain a constraint on \( \frac{t_v}{T} \). It is possible that certain radioactive nuclides are sufficiently short-lived, in that essentially all of the nuclei produced prior to some time \( t = \tilde{t} \) make no contribution to the abundance of that nuclide at \( t = T + \Delta \). If this becomes true for nuclide \( i \), we can write equation (3) as:

\[
N_i (T + \Delta) = P_i \psi(T) \Delta + e^{-\lambda_i (T+\Delta)} \int_0^T P_i \psi(t) e^{-\lambda_i (T-t)} e^{\psi(t)} dt
\]
where $T_0$ is replaced for zero as a lower limit on the integral. Define an average nucleosynthesis rate $\langle \psi \rangle_{T_0, t}$ over the interval $T_0 \leq t \leq T$ [2],

$$\langle \psi \rangle_{T_0, t} = \frac{\int_{T_0}^{T} \psi e^{\frac{\lambda_i (T-t)}{2}} dt}{\int_{T_0}^{T} e^{\frac{\lambda_i (T-t)}{2}} dt}.$$  \hspace{1cm} (23)

Substituting equation (23) into equation (22) and integrating it, we have:

$$N_i (T + \Delta) = P_i \psi (T) \Delta + \frac{P_i}{\lambda_i} e^{-\lambda_i (T - T_0)} \langle \psi \rangle_{T_0, T} \left[ 1 - e^{-\lambda_i (T - T_0)} \right].$$  \hspace{1cm} (24)

Assuming that $T - T_0$ is proportional to radionuclide half-life, therefore:

$$T - T_0 = \alpha_i t_1$$  \hspace{1cm} (25)

Substituting $T - T_0$ in equation (24), we have:

$$N_i (T + \Delta) = P_i \psi (T) \Delta + \frac{P_i}{\lambda_i} e^{-\lambda_i \alpha_i (t_1)} \langle \psi \rangle_{T_0, T} \left[ 1 - e^{-\lambda_i \alpha_i (t_1)} \right].$$  \hspace{1cm} (26)

Supposing that for two different nuclides $i$ and $j$ (half life $t_1$, $t_1$), having equal $\alpha_i$ and defining $r(i, j)$ as $\frac{\langle \psi \rangle_{T_0, T}}{\langle \psi \rangle_{T_0, T}} = \frac{\lambda_i e^{\lambda_j (T-i) \lambda_i}}{\lambda_j R(i, j)}$, with the same definition as equation (18) for $R(i, j)$, we write equation (26) for two nuclides $i$ and $j$, and obtain $r'(i, j)$ as:

$$r'(i, j) = r(i, j) A$$  \hspace{1cm} (27)

where $A$, the relating factor is:

$$A = \frac{(1 - \frac{P_i \psi \Delta}{N_i})}{\lambda_j R(i, j)}.$$  \hspace{1cm} (28)

We choose two procedures to estimate this factor and suppose that the number of nuclei produced per unit mass going into stars for both nuclides $i$ and $j$ are equal ($\psi = \psi_i = \psi_j$). In the first procedure we use the expansion of $\frac{1}{1-x}$ for $|x| < 1$, and considering equation (18) we obtain:

$$A = \sum_{n=0}^{\infty} \frac{(\Delta \psi P_j)^n}{N_j}.$$  \hspace{1cm} (29)

In the second procedure we use the first two terms of $e^{-x}$ expansion for $|x| < 1$, and calculated $A$ is:

$$A = e^{\Delta \psi \left( \frac{P_j}{N_j} - \frac{P_i}{N_i} \right)}.$$  \hspace{1cm} (30)
Finally, equation (27) will have the final form of:

\[
r'(i, j) = r(i, j) - \sum_{n=1}^{\infty} \left( \frac{\Delta \psi P}{N_j} \right)^n R^n(i, j)
\]

\[
\text{for } \left| \frac{P \Delta \psi}{N_j} \right| < 1
\]

(31-a)

and

\[
r'(i, j) = r(i, j)e^{\frac{\Delta p}{N_j}(1-R(i,j))}
\]

\[
\text{for } \left| \frac{P \Delta \psi}{N_j} \right| < \infty
\]

(31-b)

The significance of the derived equations is that they are independent of \( T \) and average half-lives as they were \( \alpha_i \) and \( \alpha_j \) dependant regarding the used method. Having \( P \psi = N\lambda/(1 - e^{-\lambda T}) \), parameters \( A, r(i, j) \) and \( r'(i, j) \) can be determined for two chronometer nuclides [8]. Using for two different nuclides \( i \) and \( j \), the relative relation for two chronometers becomes:

\[
\frac{P_i}{P_j} = \frac{N_i \lambda_i (1 - e^{-\lambda_i T})}{N_j \lambda_j (1 - e^{-\lambda_j T})}.
\]

(32)

Using the production and abundance rates of \( i \) and \( j \) at the time in which the solar system is condensed, the equilibrium time between the abundance of \( i \) and \( j \) nuclides can be determined. Substituting in equation \( P \psi = N\lambda/(1 - e^{-\lambda T}) \) and using equations (31-a) and (31-b), relations between \( r'(i, j) \) and \( r(i, j) \) are determined.

4. CONSTRAINT ON \( \frac{t_i}{T} \)

Let us now assume a set of \( m \) chronometers and label them for the longest lived chronometers by \( i=1 \), the next longest lived by \( i=2 \), and so on, to the shortest lived, labelled \( i=m \). For two chronometers \( r(2,1) \) define \( \langle \psi \rangle_{t_2,t_1} / \langle \psi \rangle_{t_1} \) [2, 5]. Averaging \( \psi e^\psi \) for nuclide 1 over all \( T \), \( \alpha_1 \) becomes \( T / \tau_{1,1} \). For \( \alpha_i = \alpha_j \), \( r(2,1) \) becomes:

\[
r(2,1) = \frac{\alpha_1 t_1 \int_{t_2}^{t_1} \frac{\alpha_1 t_1 - t_1}{t_1^2} \psi e^\psi \, dt}{\alpha_1 t_1 \int_{t_2}^{t_1} \frac{\alpha_1 t_1 - t_1}{t_1^2} \psi e^\psi \, dt}.
\]

(33)

This is calculated from equation (23) and the definition of \( r(i, j) \). To have equality between both sides of the above equation, M&S assumed that \( \psi e^\psi \) in the denominator at the right hand side is equal to one \( (\psi e^\psi = r(1,1) = 1) \) and \( \psi e^\psi = r(2,1) \) at the numerator. They improved the resolution by including more chronometers. The constrain obtained is:

\[
\psi e^\psi = r(i,1) \quad \text{for} \quad t_{i-1} \leq t \leq t_i
\]

(34)

where \( i \) runs from 1 to \( m \), the total number of chronometers, \( t_i \), is defined by \( t_i = (t_{1,1} - t_{1,1}) \) and \( \alpha_1 = T / \tau_{1,1} \). Using the above constraints, the normalized effective nucleosynthesis rate over \( t_{i-1} \leq t \leq t_i \) can be calculated [2, 5]:
\[ \phi_i = \frac{r(i,1)t_{i,1}}{T \sum_{j=1}^{m} r(k,1) \left( t_{j,k} - t_{j,k+1} \right)} \]  

and the ratio of mean time for the formation of the elements, \( t_v \) to \( T \), becomes:

\[ \frac{t_v}{T} = \frac{1}{2} \left( \frac{1}{T} \sum_{i=1}^{m} r(i,1) \left( \frac{1}{T} \sum_{j=1}^{m} r(j,1) \left( t_{j,1} - t_{j,1+1} \right) \right) \right) \]

Replacing \( aT \) instead of \( t_1 \) in equation (33) where \( 0 \leq a \leq 1 \), for two chronometers we have:

\[ \frac{t_v}{T} = \frac{(\sqrt{r(2,1)} - 1)(r(2,1) - r^2(2,1))}{\sqrt{r(2,1)}(r(2,1) - 1)^2} \]

and to obtain \( T \), substitute \( \phi_i \) and \( \phi_j \) from equation (21).

5. RESULTS AND CONCLUSIONS

Suppose that we have two chronometer nuclides and we use \( i = 1 \) for \( ^{232}Th \) and \( j = 2 \) for \( ^{238}U \), with decay constant \( 4.95 \times 10^{-11} \text{yr}^{-1} \) and \( 1.5512 \times 10^{-10} \text{yr}^{-1} \), respectively. From the M&S data tables \( R(2,1) = 1.49(-0.21, +0.30) \).

For the extreme of \( t_v/T \) we require the extreme of \( r'(i,j) \). If we use production and abundance rate of Thorium / Uranium from Schramm & Wasserburg [3] and the M&S papers [2], and substitute in equation \( P\psi = N\lambda/(1-e^{-\lambda t}) \), we will have an exponential equation for \( T \) produces two values for \( T \) (Fig. 1). It is shown that there are two points for the production rate equilibrium, \( T_1 = 0 \text{Gyr} \) and \( T_2 = 7.0958\text{Gyr} \). Fig. 1 shows that \( R(Th, U) \) approaches zero in two cases: The first is in the case that production value for Thorium becomes much less than the Uranium’s production value and can be neglected. The second case occurs at the condensation era of the Solar System in which the Thorium abundance becomes much more than the Uranium abundance. Substituting \( T_2 \) in equation \( P\psi = N\lambda/(1-e^{-\lambda t}) \) gives us \( (P\psi/N) \) and consequently from equations (31-a) and (31-b), the relation between \( r'(i,j) \) and \( r(i,j) \) are determined, \( r'(i,j) = r(i,j) = 1 \), \( r'_{\text{max}}(2,1) = 2.46 \) and \( r'_{\text{min}}(2,1) = 0.57 \). Equation (37) gives us the following range for \( \frac{t_v}{T} \):

\[ 0.43 \leq \frac{t_v}{T} \leq 0.61 \]

Using the above range and equations (21) and (35), the constraint on \( T \) will be

\[ 7.96 \text{Gyr} \leq T \leq 18.45 \text{Gyr} \]

Considering the age of the solar system [9], the constraints on galaxy age become:

\[ 12.5 \text{Gyr} \leq T_{\text{gal}} \leq 23.0 \text{Gyr} \]

Finally, adding the galactic condensation age (\( \delta=1.5\pm0.5 \text{Gyr} \)), our calculated constraint on the age of the universe will be:
M&S determined the galaxy age to be between 8.7 Gyr to 28.1 Gyr. According to the obtained results, the range of constraints for the age of the universe is shorter than the M&S and our modifications on the model have reduced the interval for the galaxy age 83% and close to being concordant with the other available methods. Since uranium has only been observed in a few stars, several groups have employed the Th/U ratio to determine the chronometric age of CS31082-001 and the age of the galaxy [10]. Cayrel et al. used this method and determined \( 12.5 \pm 3 \) (Gyr) for the age [11]. Hill et al. evaluated this age \( 14.0 \pm 2.4 \) (Gyr) [12]. Schatz et al. got to the age of \( 15.5 \pm 3.2 \) (Gyr) [13] and Wanjo et al. determined \( 14.1 \pm 2.5 \) (Gyr) for the age [14]. Dauphas used Th/U in meteorites in conjunction with the observation of halo stars and coupled with a chemical evolution method to determine the age of the galaxy \( 14.5^{+2.8}_{-2.2} \) (Gyr) [15]. Tegmark et al. used Wilkinson Microwave Anisotropy Probe and combine that with the results from the Sloan Digital Sky Survey and estimated the age \( 14.1^{+1.0}_{-1.9} \) (Gyr) for the galaxy [16]. Krauss and Chaboyer used cluster results based upon main sequence turn off ages and evaluated the age \( 12.5^{+3.3}_{-2.2} \) (Gyr) [17]. Jimenez et al. with the inclusion of CMB data arrived at the age \( 12.6^{+3.4}_{-2.2} \) (Gyr) [18]. In one case we have enough data to use all the terms of equations (20), and having the abundance and production ratio of Th/U more accurately limited than at present, improving the certainty of the correct Th/U abundance ratio (i.e. understanding meteoritic) and of the Th/U production ratio in the r-process, our result from nuclear chronometrical techniques can be more limited and closer to other astrophysical methods.

Fig. 1. Production rate equilibrium times for Th/U ratio. It is shown that there were two points for the production rate equilibrium and after at least 3.2 Gyr of nucleosynthesis in stars and supernovae, the production to abundance ratio of Th/U achieved its maximum point and then decreases. At the end of the chemical evolution period 7.09 Gyr and time interval \( \Delta \), before the condensation of the solar system, the Thorium/Uranium production ratio becomes much less than their abundance ratio. Since the decay constants ratio for Thorium /Uranium have a fixed value, \( \lambda_U / \lambda_{Th} = 3.1337 \), the peak value shows that, the production ratio for Th/U, at least after 3.2 Gyr of the nucleosynthesis era, remains less than the abundance ratio. Infinite minus slope of \( R(\text{Th},\text{U}) \) shows that by the passing of time this reduction will continue.

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