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On generalized statistical convergence in random 2-normed space

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Abstract

In this paper, we shall define and study the concept of λ-statistical convergence and λ-statistical Cauchy in random 2-normed space. We also introduce the concept of λ-statistical completeness which would provide a more general frame work to study the completeness in random 2-normed space. Furthermore, we also prove some new results.

Keywords: Statistical convergence; λ-statistical convergence; t-norm; 2-norm; random 2-normed space

1. Introduction

In many branches of science and engineering we often come across different types of sequences and certainly there are situations where either the idea of ordinary convergence does not work or the underlying space does not serve our purpose. So to ideal with such situations some new types of measures must be introduced which can provide a better tool and a suitable frame work.

The probabilistic metric space was studied by Menger [1], which is an interesting and important generalization of the notion of a metric space. The theory of probabilistic normed (or metric) spaces was initiated and developed in [2-6] and, it was further extended to random/probabilistic 2-normed space by Golet [7] using the concept of 2-norm which is defined by Gahler [8, 9] and Gürdal and Pehlivan [10] studied statistical convergence in 2-normed spaces. Also, statistical convergence in 2-Banach spaces was studied by Gürdal and Pehlivan in [11].

The notion of statistical convergence was introduced by Fast [12] and Schoenberg [13] independently. Numerous developments have been made in this area after the works of Salat [14], and Fridy [15]. Over the years and under different names statistical convergence has been discussed in the theory of Fourier analysis, ergodic theory and number theory. Quite recently in [16], λ-statistical convergence was studied for double sequence spaces in probabilistic normed space by Savas and Mohiuddine.

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The notion of statistical convergence depends on the density of subsets of N, the set of natural numbers. Let K be a subset of N. Then the asymptotic density of K denoted by $\delta(K)$, is defined as

$$\delta(K) = lim_{n\to\infty}\frac{1}{n}\left|\left\{k \le n : k \in N\right\}\right|,$$

where the vertical bars denote the cardinality of the enclosed set.

A single sequence $x = (x_k)$ is said to be statistically convergent to l if for every $\varepsilon > 0$ the set $K(\varepsilon) = \left\{k \le n : \left|x_k - l\right| \ge \varepsilon\right\}$ has asymptotic density zero, i.e.

$$lim_{n\to\infty}\frac{1}{n}\left|\left\{k \le n : \left|x_k - l\right| \ge \varepsilon\right\}\right| = 0.$$

In this case we write $S - lim\, x = l$ or $x_k \to l(S)$, (see [12], [15]).

2. Definitions and preliminaries

Definition 2.1. A function $f : \mathbb{R} \to \mathbb{R}_o^{\ +}$ is called distribution function if it is non-decreasing and left continuous with $inf_{t\in\mathbb{R}}\, f(t) = 0$ and $sup_{t\in\mathbb{R}}\, f(t) = 1$. By D^+, we denote the set of all distribution functions such that $f(0) = 0$. If $a \in \mathbb{R}_o^{\ +}$, then $H_a \in D^+$, where

$$H_a(t)=\begin{cases}1, & if\ t>a;\\ 0, & if\ t\le a.\end{cases}$$

It is obvious that $H_o \ge f$ for all $f \in D^+$.

A *t*-norm is a continuous mapping $*: [0,1]\times[0,1]\to[0,1]$ such that $([0,1],*)$ is abelian monoid with unit one and $c*d \ge a*b$ if $c\ge a$ and $d \ge b$ for all $a,b,c,d\in[0,1]$. A triangle function τ is a binary operation on D^+, which is commutative, associative and $\tau(f,H_o)=f$ for every $f\in D^+$.

In [8], Gahler introduced the following concept of 2-normed space.

Definition 2.2. A 2-normed space is a pair $(X,\|.,.\|)$, where *X* is a linear space of greater than one and $\|.,.\|: XxX\to R$ such that

(1) $\|x_1,x_2\|=0$ if and only if x_1,x_2 are linearly dependent,

(2) $\|x_1,x_2\|$ is invariant under the permutation,

(3) $\|\alpha x_1,x_2\|=|\alpha|\|x_1,x_2\|$, for any $\alpha\in\mathbb{R}$, and

(4) $\|x_1+\bar{x},x_2\|\le\|x_1,x_2\|+\|\bar{x},x_2\|,$ *for all* $\bar{x},x_1,x_2\in X$.

A trivial example of a 2-normed space is $X=\mathbb{R}^2$, equipped with the Euclidean 2-norm $\|x_1,x_2\|_E$ = the area of the parallelogram spanned by the vectors x_1,x_2 which may be given expicitly by the formula

$$\|x_1,x_2\|_E = |\det(x_{ij})| = abs(\det(<x_i,x_j>))$$

where $x_i=(x_{i1},x_{i2})\in\mathbb{R}^2$ for each $i=1,2$.

Recently, Golet [7] used the idea of 2-normed space to define the random 2-normed space.

Definition 2.3. Let *X* be a linear space of dimension $d>1$ (d may be infinite), τ a triangle, and $\mathscr{F}: X\times X\to D^+$. Then $\mathscr{F}$ is called a probabilistic 2-norm and $(X,\mathscr{F},\tau)$ a *probabilistic 2-normed space* if the following conditions are satisfied:

(P2N1) $\mathscr{F}(x,y;t)=H_o(t)$ if *x* and *y* are linearly dependent, where $\mathscr{F}(x,y;t)$ denotes the value of $\mathscr{F}(x,y)$ at $t\in\mathbb{R}$.

(P2N2) $\mathscr{F}(x,y;t)\ne H_o(t)$ if *x* and *y* are linearly independent,

(P2N3) $\mathscr{F}(x,y;t)=\mathscr{F}(y,x;t)$, for all $x,y\in X$,

(P2N4) $\mathscr{F}(\alpha x,y;t)=\mathscr{F}(y,x;\frac{t}{|\alpha|})$, for every $t>0$, $\alpha\ne 0$ and $x,y\in X$,

(P2N5) $\mathscr{F}(x+y,z;t)\ge\tau(\mathscr{F}(x,z;t),\mathscr{F}(y,z;t))$, whenever $x,y,z\in X$.

If (P2N5) is replaced by (P2N6) $\mathscr{F}(x+y,z;t_1+t_2)\ge\mathscr{F}(x,z;t_1)*\mathscr{F}(y,z;t_2)$ for all $x,y,z\in X$ and $t_1+t_2\in\mathbb{R}_o^{\ +}$; then $(X,\mathscr{F},*)$ is called a *random 2-normed space* (for short, R2NS).

Remark 2.1. Note that every 2-normed space $(X,\|.,.\|)$ can be made a random 2-normed space in a natural way, by setting

$\mathscr{F}(x,y;t)\ge H_o(t-\|x,y\|)$, for every $x,y\in X$, $t>0$ and $a*b=\min\{a,b\}$, $a,b\in[0,1]$;

$\mathscr{F}(x,y;t)=\frac{t}{t+\|x,y\|}$, for every $x,y\in X$, $t>0$ and $a*b=ab$, $a,b\in[0,1]$.

We have

Definition 2.4. A sequence $x=(x_k)$ in a random 2-normed space $(X,\mathscr{F},*)$ is said to be convergent (or $\mathscr{F}$-convergent) to $l\in X$ with respect to $\mathscr{F}$ if for every $t>0$, $\varepsilon\in(0,1)$, there exists an positive integer N such that $\mathscr{F}(x_k-l,z;t)>1-\varepsilon$, whenever $k\ge N$ and for non zero $z\in X$. In this case we write $\mathscr{F}\text{-}\lim_k x_k=l$, and l is called the $\mathscr{F}$-limit of $x=(x_k)$.

Definition 2.5. A sequence $x=(x_k)$ in a random 2-normed space $(X,\mathscr{F},*)$ is said to be Cauchy with respect to $\mathscr{F}$ if for each $t>0$, $\varepsilon\in(0,1)$, there exists an positive integer $N=N(\varepsilon)$ such that $\mathscr{F}(x_k-x_m,z;t)>1-\varepsilon$, whenever $k,m\ge N$ and for non zero $z\in X$.

In [17], Mursaleen studied the concept of statistical convergence of sequences in random 2-normed space.

Definition 2.6. [17]. A sequence $x=(x_k)$ in a random 2-normed space $(X,\mathscr{F},*)$ is said to be *statistical-convergent* (or S^{R2N}*-convergent*) to some $l\in X$ with respect to $\mathscr{F}$ if for each $t>0$, $\varepsilon\in(0,1)$, and for non zero $z\in X$ such that

$$\delta\left(\left\{k \in N : \mathscr{F}\left(x_k - l, z; t\right) \le 1 - \varepsilon\right\}\right) = 0.$$

In other words, we can write the sequence (x_k) statistical converges to l in random 2-normed space $(X, \mathscr{F}, *)$ if

$$lim_{m\to\infty} \frac{1}{m} \left| \left\{k \le m : \mathscr{F}\left(x_k - l, z; t\right) \le 1 - \varepsilon\right\} \right| = 0 \cdot$$

or equivalently

$$\delta\left(\left\{k \in N : \mathscr{F}\left(x_k - l, z; t\right) \ge 1 - \varepsilon\right\}\right) = 1,$$

i.e.

$$S - lim_{k\to\infty} \mathscr{F}\left(x_k - l, z; \varepsilon\right) = 1.$$

In this case we write $S^{R2N} - \lim x = l$ and l is called the S^{R2N} – limit of *x*. Let $S^{R2N}(X)$ denote the set of all statistical convergent sequences in random 2-normed space $(X, \mathscr{F}, *)$.

Quite recently, Mohiuddine and Aiyub [18] introduced the concept of lacunary statistical convergence in random 2-normed space.

In this paper we study λ-statistical convergence in random 2-normed space, which is an interesting new idea. We show that some properties λ-statistical convergence of real numbers also hold for sequences in random 2-normed spaces. We establish some relations related to statistical convergent and λ-statistical convergent sequences in random 2-normed spaces.

3. λ-Statistical convergence in random 2-normed space

Now we shall define λ-statistical convergence in random 2-normed space $(X, \mathscr{F}, *)$. Also, we get some basic properties of this notion in random 2-normed space.

Let $\lambda = (\lambda_n)$ be a non-decreasing sequence of positive numbers tending to ∞ such that

$$\lambda_{n+1} \le \lambda_n + 1, \lambda_1 = 1.$$

Let $K \subset \mathbb{N}$. The number $\delta_\lambda(K)$ if

$$lim_n \tfrac{1}{\lambda_n} \left| \left\{n - \lambda_n + 1 \le k \le n : k \in K\right\} \right|$$

is said to be λ-density of K. If $\lambda_n = n$ for every *n*, then λ-density is reduced to the asymptotic density.

Definition 3.2. A sequence $x = (x_k)$ is said to be λ-statistical convergence or S_λ - convergent to the number l if for every $\varepsilon > 0$, the set $N(\varepsilon)$ has λ-density zero, where

$$N(\varepsilon) = \left\{k \in I_n : | x_k - l | \ge \varepsilon\right\},$$

and $I_n = \left[n - \lambda_n + 1, n\right]$. In this case, we write $st_\lambda - lim\, x = L$, (for details see [19]).

Now we are ready to define λ-statistically convergent in random 2-normed space.

Definition 3.3. A sequence $x = (x_k)$ in a random 2-normed space $(X, \mathscr{F}, *)$ is said to be λ-*statistically convergent* or S_λ-*convergent* to $l \in X$ with respect to $\mathscr{F}$ if for every $t > 0, \varepsilon \in (0,1)$, and for non zero $z \in X$ such that

$$\delta_\lambda\left(\left\{k \in N : \mathscr{F}\left(x_k - l, z; t\right) \le 1 - \varepsilon\right\}\right) = 0$$

or equivalently

$$\delta_\lambda\left(\left\{k \in N : \mathscr{F}\left(x_k - l, z; t\right) > 1 - \varepsilon\right\}\right) = 1.$$

In this case we write $S_\lambda^{R2N} - lim\, x = l$ or $x_k \to l(S_\lambda^{R2N})$ and

$$S_\lambda^{R2N}(X) = \left\{x = (x_k) : \exists l \in \mathbb{R}, S_\lambda^{R2N} - lim\, x = l\right\}.$$

Let $S_\lambda^{R2N}(X)$ denote the set of all λ-statistical convergent sequences in random 2-normed space $(X, \mathscr{F}, *)$.

If $\lambda_n = n$ for every n then λ-statistical convergent sequences in random 2-normed space $(X, \mathscr{F}, *)$ reduce to statistical convergent sequences in random 2-normed space $(X, \mathscr{F}, *)$.
The above definition immediately implies the following Lemma.

Lemma 3.1. Let $(X, \mathscr{F}, *)$ be a random 2-normed space. If $x = (x_k)$ is a sequence in *X*, then for every

$t>0$, $\varepsilon\in(0,1)$, and for nonzero $z\in X$, then the following statements are equivalent.

(i) $S^{R2N}-\lim_{k\to\infty}x_k=l$.

(ii) $\delta_\lambda\left(\left\{k\in I_m:\mathscr{F}\left(x_k-l,z;t\right)\le 1-\varepsilon\right\}\right)=0$.

(iii) $\delta_\lambda\left(\left\{k\in I_m:\mathscr{F}\left(x_k-l,z;t\right)>1-\varepsilon\right\}\right)=1$.

(iv) $S_\lambda-\lim_{k\to\infty}\mathscr{F}\left(x_k-l,z;t\right)=1$.

Theorem 3.2. Let $(X,\mathscr{F},*)$ be a random 2-normed space. If $x=(x_k)$ is a sequence in X such that $S_\lambda^{R2N}-\lim x_k=l$ exists, then it is unique.

Proof: Suppose that

$$S_\lambda^{R2N}-\lim_{k\to\infty}x_k=l_1,\ S_\lambda^{R2N}-\lim_{k\to\infty}x_k=l_2.$$

Given $\varepsilon>0$ and $p>0$ such that

$$(1-p)*(1-p)>1-\varepsilon. \tag{3.1}$$

Then, we define the following sets as: for any $t>0$ and non zero $z\in X$,

$$K_1(p,t)=\left\{k\in I_n:\mathscr{F}(x_k-l_1,z;\tfrac{t}{2})\le 1-p\right\};$$

$$K_2(p,t)=\left\{k\in I_n:\mathscr{F}(x_k-l_2,z;\tfrac{t}{2})\le 1-p\right\}.$$

Since

$$S_\lambda^{R2N}-\lim_{k\to\infty}x_k=l_1,\ S_\lambda^{R2N}-\lim_{k\to\infty}x_k=l_2,$$

we have

$$\delta_\lambda\left(K_1(p,t)\right)=0 \text{ and } \delta_\lambda\left(K_2(p,t)\right)=0$$

for all $t>0$.

Now let $K(p,t)=K_1(p,t)\cup K_2(p,t)$, then it is easy to see that $\delta_\lambda\left(K(p,t)\right)=0$ which implies $\delta_\lambda\left(K^c(p,t)\right)=1$.

Now, if $k\in K^c(p,t)$ then we have

$$\mathscr{F}(l_1-l_2,z;t)\ge\mathscr{F}(x_k-l_1,z;\tfrac{t}{2})$$
$$*\mathscr{F}(x_k-l_2,z;\tfrac{t}{2})>(1-p)*(1-p).$$

It follows by (3.1) that

$$\mathscr{F}(l_1-l_2,z;t)>1-\varepsilon.$$

Since $\varepsilon>0$ was arbitrary, we get $\mathscr{F}(l_1-l_2,z;t)=0$ for all $t>0$ and non zero $z\in X$ which yields $l_1=l_2$. This completes the proof of the theorem.
We have,

Theorem 3.3. Let $(X,\mathscr{F},*)$ be a random 2-normed space, and $x=(x_k)$ and $y=(y_k)$ be two sequences in X.

(a) If $S_\lambda^{R2N}-\lim x_k=l$ and $c(\ne 0)\in\mathbb{R}$, then $S_\lambda^{R2N}-\lim cx_k=cl$.

(b) If $S_\lambda^{R2N}-\lim x_k=l_1$ and $S_\lambda^{R2N}-\lim y_k=l_2$, then $S_\lambda^{R2N}-\lim(x_k+y_k)=l_1+l_2$.

Proof is easy and omitted.

Theorem 3.4. Let $(X,\mathscr{F},*)$ be a random 2-normed space. If $x=(x_k)$ be a sequence in X such that $\mathscr{F}-\lim x_k=l$, then $S_\lambda^{R2N}-\lim x_k=l$. But the converse need not be true in general.

Proof: Let $\mathscr{F}-\lim x_k=l$. Then for every $\varepsilon>0$, $t>0$ and non zero $z\in X$, there is a positive integer N such that

$$\mathscr{F}(x_k-l,z;t)>(1-\varepsilon)$$

for all $k\ge N$. Since the set

$$K(\varepsilon,t)=\left\{k\in I_n:\mathscr{F}(x_k-l,z;t)\le 1-\varepsilon\right\}$$

has at most finitely many terms. Since every finite subset of N has density zero, we have $\delta_\lambda(K(\varepsilon,t))=0$. This shows that $S_\lambda^{R2N}-\lim x_k=l$.

This completes the proof of the theorem.

The following example shows that the converse need not be true.

Example 3.6. Let $X=\mathbb{R}^2$, with the 2-norm $\|x,z\|=|x_1z_2-x_2z_1|$, $x=(x_1,x_2)$, $z=(z_1,z_2)$ and $a*b=ab$ for all $a,b\in[0,1]$. Let $\mathscr{F}(x,z;t)=\frac{t}{t+\|x,z\|}$, for all $x,z\in X$, $z\ne 0$, and $t>0$. We write a sequence $x=(x_k)$ by

$$x_k=\begin{cases}(k,0), & \text{if } n-[\sqrt{\lambda_n}]+1\le k\le n,\\ (0,0), & \text{otherwise.}\end{cases}$$

It is easy to observe that, this sequence is $S_\lambda^{R2N} - lim\, x_k = 0,$ while it is obvious that $\mathscr{F} - lim\, x_k \neq 0.$

Now we are in a position to present the following.

Theorem 3.7. Let $(X, \mathscr{F}, *)$ be a random 2-normed space. If $x = (x_k)$ be a sequence in *X*, then $S_\lambda^{R2N} - lim\, x_k = l$ if and only if there exists a subset $K = \{k_1 < k_2, ...\} \subseteq \mathbb{N}$ such that $\delta_\lambda(K) = 1$ and $\mathscr{F} - \lim_{n\to\infty} x_{k_n} = l$.

Proof: Necesssity: Suppose that $S_\lambda^{R2N} - lim\, x_k = l$. Then for any $t > 0$, $p = 1, 2, 3, ...$ and non zero $z \in X$, let

$$A(p,t) = \left\{k \in I_n : \mathscr{F}(x_k - l, z; t) > 1 - \tfrac{1}{p}\right\}$$

and

$K(p,t) = \left\{k \in I_n : \mathscr{F}(x_k - l, z; t) \leq 1 - \tfrac{1}{p}\right\}.$

Since $S_\lambda^{R2N} - lim\, x_k = l$ it follows that $\delta_\lambda(K(p,t)) = 0$.

Now for $t > 0$, $p = 1,2,3,...$, we have that

$$A(p,t) \supset A(p+1,t)$$

and

$$\delta_\lambda(A(p,t)) = 1. \tag{3.2}$$

Now we have to show that, for $k \in A(p,t)$, $\mathscr{F} - lim\, x_k = l.$ Suppose that for some $k \in A(p,t)$, (x_k) is not convergent to l with respect to $\mathscr{F}$. Then there exists some $s > 0$ and a positive integer k_0 such that

$$\{k \in I_n : \mathscr{F}(x_k - l, z; t) \leq 1 - s\}$$

for all $k \geq k_0$. Let

$$\mathscr{F}(x_k - l, z; t) > 1 - s$$

for all $k < k_0$. Then

$$\delta_\lambda\{\ k \in I_n : \mathscr{F}(x_k - l, z; t) > 1 - s\ \} = 0.$$

Since

$$s > \tfrac{1}{p},\ p = 1,2,3,...\ .$$

we have

$$\delta_\lambda(A(p,t)) = 0,$$

which contradicts (3.2) as $\delta_\lambda(A(p,t)) = 1$. Hence $\mathscr{F} - \lim x_k = l$.

Sufficiency: Suppose that there exists a subset $K = \{k_1 < k_2 < ...\} \subseteq \mathbb{N}$ such that $\delta_\lambda(K) = 1$ and $\mathscr{F} - \lim_{n\to\infty} x_{k_n} = l$, i.e. there exists $N \in \mathbb{N}$ such that for every $s > 0$, $t > 0$ and non zero $z \in X$

$$\mathscr{F}(x_k - l, z; t) > 1 - s$$

for all $k \geq N$. If we take

$$K(s,t) = \{k \in I_n : \mathscr{F}(x_k - l, z; t) \leq 1 - s\}$$

then it is easy to see that

$$K(s,t) \subset \mathbb{N} - \{N_{k+1}, N_{k+2}, ...\}$$

and therefore

$$\delta_\lambda(K(s,t)) \leq 1 - 1.$$

This completes the proof of the theorem.

Now we establish the Cauchy convergence criteria in random 2-normed spaces.

Definition 3.4. A sequence $x = (x_k)$ in a random 2-normed space $(X, \mathscr{F}, *)$ is said to be λ-statistical Cauchy with respect to $\mathscr{F}$ if for every t>0, $\varepsilon \in (0,1)$, and non zero $z \in X$ there exists a positive integer $N = N(\varepsilon)$ such that

$$\delta_\lambda\left(\{k \in I_n : \mathscr{F}(x_k - x_N, z; t) \leq 1 - \varepsilon\}\right) = 0.$$

or equivalently

$$\delta_\lambda\left(\{k \in I_n : \mathscr{F}(x_k - x_N, z; t) > 1 - \varepsilon\}\right) = 1.$$

Theorem 3.8. Let $(X, \mathscr{F}, *)$ be a random 2-normed space. Then a sequence (x_k) is $\lambda-$statistically convergent if and only if it is $\lambda-$statistically Cauchy.

Proof: Let (x_k) be a $\lambda-$statistically convergent to l with respect to random 2-normed space, i.e. $S_\lambda^{R2N} - lim\, x_k = l.$ For a given $\varepsilon > 0$, choose $p > 0$ such that (3.1) is satisfied. For $t > 0$ and for non zero $z \in X$ define

$$A(p,t)=\{k\in I_n:\mathscr{F}(x_k-l,z;\tfrac{t}{2})\le 1-p\}.$$

Hence

$$A^c(p,t)=\{k\in I_n:\mathscr{F}(x_k-l,z;\tfrac{t}{2})>1-p\}.$$

Since $S_\lambda^{R2N}-\lim x_k=l$ it follows that $\delta_\lambda(A(p,t))=0$ and consequently $\delta_\lambda(A^c(p,t))=1$. Let $q\in A^c(p,t)$. Then

$$\mathscr{F}(x_q-l,z;\tfrac{t}{2})>1-p. \tag{3.3}$$

If we take

$$B(\varepsilon,t)=\{k\in I_n:\mathscr{F}(x_k-x_q,z;t)\le 1-\varepsilon\},$$

we need to show that $B(\varepsilon,t)\subseteq A(p,t)$.

Let $k\in B(\varepsilon,t)\setminus A^c(p,t)$, then for non zero $z\in X$ we have

$$\mathscr{F}(x_k-x_q,z;t)\le 1-\varepsilon \text{ and} \tag{3.4}$$

$$\mathscr{F}(x_k-l,z;\tfrac{t}{2})>1-p.$$

Now from (3.1), (3.3) and (3.4) we get

$$1-\varepsilon\ge\mathscr{F}(x_k-x_q,z;t)\ge\mathscr{F}(x_k-l,z;\tfrac{t}{2})*\mathscr{F}(x_q-l,z;\tfrac{t}{2})$$

$$>(1-p)*(1-p)>(1-\varepsilon)$$

which is not possible. Hence $B(\varepsilon,t)\subset A(p,t)$. Since $\delta_\lambda(A(p,t))=0$, it follows that $\delta_\lambda(B(\varepsilon,t))=0$. This shows that (x_k) is $\lambda-$statistically Cauchy.

Conversely, suppose that (x_k) is $\lambda-$statistically Cauchy but not $\lambda-$statistically convergent with respect to $\mathscr{F}$. Then for a given $\varepsilon>0$, $t>0$ and for non zero $z\in X$, there exists a positive integer $N=N(\varepsilon)$ such that

$$E(\varepsilon,t)=\{k\in I_n:\mathscr{F}(x_k-x_N,z;t)\le 1-\varepsilon\}$$

then

$$\delta_\lambda(E(\varepsilon,t))=0$$

and evidently

$$\delta_\lambda(E^c(\varepsilon,t))=1. \tag{3.5}$$

For $t>0$, choose $p>0$ such that (3.1) is satisfied, and we take

$$B(p,t)=\{k\in I_n:\mathscr{F}(x_k-l,z;\tfrac{t}{2})>1-p\}.$$

If $N\in B(p,t)$, then $\mathscr{F}(x_N-l,z;\tfrac{t}{2})>1-p$. Since

$$\mathscr{F}(x_k-x_N,z;t)\ge\mathscr{F}(x_k-l,z;\tfrac{t}{2})$$
$$*\mathscr{F}(x_N-l,z;\tfrac{t}{2})>(1-p)*(1-p)>1-\varepsilon,$$

then we have

$$\delta_\lambda\left(\{k\in I_n:\mathscr{F}(x_k-x_N,z;t)>1-\varepsilon\}\right)=0$$

i.e., $\delta_\lambda(E^c(\varepsilon,t))=0$, which contradicts (3.5) as $\delta_\lambda(E^c(\varepsilon,t))=1$. Hence (x_k) is $\lambda-$statistically convergent. This completes the proof of the theorem

Definition 3.5. A random 2-normed space $(X,\mathscr{F},*)$ is said to be *complete* if every Cauchy sequence is convergent in $(X,\mathscr{F},*)$.

We define the following definition in random 2-normed space as a consequence of the Theorem 3.8.

Definition 3.6. A random 2-normed space $(X,\mathscr{F},*)$ is said to be $S_\lambda-$complete if every $S_\lambda-$Cauchy sequence is $S_\lambda-$convergent in $(X,\mathscr{F},*)$.

Theorem 3.9. Every random 2-normed space $(X,\mathscr{F},*)$ is $S_\lambda-$complete but not complete in general.

Proof: First part of the proof of the theorem follows from the Theorem 3.8. To observe that random 2-normed space $(X,\mathscr{F},*)$ is not complete in general, we consider the following example:

Example 3.10. Let $X=(0,1]\times(0,1]$ with the 2-norm $\|x,z\|=|x_1z_2-x_2z_1|$, $x=(x_1,x_2)$, $z=(z_1,z_2)$ and $a*b=ab$ for all $a,b\in[0,1]$. Let $\mathscr{F}(x,z;t)=\frac{t}{t+\|x,z\|}$, for all $x,z\in X$, $z\neq 0$, and $t>0$. Then $(X,\mathscr{F},*)$ is a random 2-normed space but not complete, since the sequence $(\frac{1}{n},\frac{1}{m})$ is Cauchy with respect to $\mathscr{F}$ but not convergent. This completes the proof of the theorem.

We get the following Corollary by combining the theorems 3.7, 3.8 and 3.9.

Corollary 3.11. Let $(X,\mathscr{F},*)$ be a random 2-normed space and $x=(x_k)$ be a sequence in X. Then the following statements are equivalent:

(a) x is λ-statistically convergent with respect to the random 2-normed spaces.

(b) x is λ-statistically Cauchy with respect to the random 2-normed spaces.

(c) random 2-normed space $(X,\mathscr{F},*)$ is $S_\lambda - complete$.

(d) there exists a subset $K=\{k_n : k_1<k_2 ...\}\subseteq \mathbb{N}$ such that $\delta_\lambda(K)=1$ and the subsequence (x_{k_n}) is $S_\lambda -$Cauchy with respect to the random 2-normed spaces.

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