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On generalized conformally recurrent Kaehlerian Weyl spaces

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Abstract

In this study, $2n$-dimensional $(n>2)$ generalized conformally recurrent Kaehlerian Weyl spaces and generalized conharmonicaly recurrent Kaehlerian Weyl spaces are defined. It is proved that a Kaehlerian Weyl space is generalized conformally recurrent if and only if it is generalized recurrent.Also, it is shown that a Kaehlerian Weyl space will be generalized recurrent if and only if it is generalized conharmonically recurrent.

Keywords*:* Kaehlerian Weyl space, recurrent Weyl space, generalized recurrency, conformally recurrency, conharmonic recurrency

1. Introduction

In the review paper "Weyl Geometry in Late 20th Century Physics" by E. Scholz, the physical concepts in Weylian geometry has been widely reviewed (see [1] and references therein). H. Weyl first introduced a gauge invariant theory to unify gravity and electromagnetic theories in 1918. This theory is not acceptable as a unified theory since the electromangetic potential does not couple to spinor which is essential for the electromagnetic theory. This does not mean that Weyl geometry has a physical meaning as well in different theories and, in the second part of the 20th century, the Weyl geometry has been studied in different research fields of physics such as: quantum mechanics [2], particle physics [3], gravity [4] and scale invariant cosmology.

In [4], spaces with complex structures are considered in Weyl geometry and geodesics of the spacetime are studied. They consider scalar-flat Kaehler metrics and hypercomplex structures and obtain shear-free congruences. Presence of this type congruence means that a pair of coupled monopole like solution is equivalent to Einstein-Weyl equation. In quantum mechanics, discovery of phenomenon such as Berry phase (geometric phase), adiabatic transition probability in two level quantum system caused physicists to consider the geometry of quantum mechanics more than Riemannian structure of Hilbert space and therefore, they consider that physical state for the Berry phase is isomorphic to Kahlerian space and physical state for adiabatic transition may be related to Kaehlerian-Weyl space [5]. It may also be worthwhile to study Dirac equation in the background of these type of spaces to understand the effects of torsion on spinor fields and to study gravitational monopoles as an application of Kaehlerian-Weyl spaces.

Despite the unified theory of Weyl not being acceptable as physical theory, it introduced a beautiful theory in differential geometry. The mathematics of the theory is a generalization of the Riemannian geometry and the connection is an instructive example of non-metric connections.

In this work, some generalized Kehlerian Weyl spaces are considered and some structures on these spaces are examined.

A differentiable manifold of dimension n having a conformal metric tensor g and a symmetric connection ∇ satisfying the compatibility condition

$$\nabla g = 2(T\otimes g) \tag{1}$$

where T is a 1-form (complementary covector field) is called a Weyl space which is denoted by $W_n(g,T)$. Under the renormalization

$$\bar{g} = \lambda^2 g \tag{2}$$

of the metric tensor g, T is transformed by the law

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$$\bar{T} = T + d\ln\lambda, \quad (3)$$

where λ is a scalar function defined on W_n .

An object A defined on $W_n(g,T)$ is called a satellite of g of weight p if it admits a transformation of the form

$$\bar{A} = \lambda^p A \quad (4)$$

under the renormalization (2) of g , [6-9].

The prolonged covariant derivatives of the satellite A of the tensor g_{ij}, with weight p is defined in [6]

$$\dot{\nabla}_k A = \nabla_k A - pT_k A. \quad (5)$$

Writing (1) in local coordinates we find

$\partial_k g_{ij} - g_{hj}\Gamma^h_{ik} - g_{ih}\Gamma^h_{jk} - 2T_k g_{ij} = 0, \qquad \partial_k = \dfrac{\partial}{\partial x^k}$,

and using (5) we obtain

$$\dot{\nabla}_k g_{ij} = 0$$

where Γ^i_{kl} are the coefficients of the Weyl connection given by

$$\Gamma^i{}_{kl} = \begin{Bmatrix} i \\ kl \end{Bmatrix} - g^{im}(g_{mk}T_l + g_{ml}T_k - g_{kl}T_m) \quad (6)$$

and $\begin{Bmatrix} i \\ kl \end{Bmatrix}$ being the coefficients of the metric connection defined by

$$\begin{Bmatrix} i \\ kl \end{Bmatrix} = \frac{1}{2} g^{im}(\partial_k g_{ml} + \partial_l g_{km} - \partial_m g_{kl}). \quad (7)$$

We note that the prolonged covariant differentiation preserves the weights of the satellites.

A Kaehlerian Weyl space denoted by KW_n is an n-dimensional ($n = 2m$) space with an almost complex structure $F_i^{\ j}$ satisfying

$$F_i^{\ j} F_j^{\ k} = -\delta_i^k \quad (8)$$

$$g_{ij} F_h^i F_k^j = g_{hk} \quad (9)$$

$$\dot{\nabla}_k F_i^{\ j} = 0, \qquad (for\,all\;\; i,j,k) \quad (10)$$

$$F_{ij} = g_{jk} F_i^k = -F_{ji}, \quad (11)$$

$$F^{ij} = g^{ih} F_h^j = -F^{ji}. \quad (12)$$

The tensors F_{ij} and F^{ij} are of weight 2 and -2, respectively [9] .

The mixed curvature tensor R^i_{jkl} and the covariant curvature tensor R_{ijkl} of $W_n(g,T)$ are given respectively,

$$R^i_{jkl} = \frac{\partial}{\partial x^l}\Gamma^i_{jk} - \frac{\partial}{\partial x^k}\Gamma^i_{jl} + \Gamma^i_{hl}\Gamma^h_{jk} - \Gamma^i_{hk}\Gamma^h_{jl} \quad (13)$$

and $R_{ijkl} = g_{ih} R^h_{jkl}$ where Γ^i_{jk}'s are defined in (6) .

The Ricci tensor and the scalar curvature of $W_n(g,T)$ are defined by

$$R^a_{ija} = R_{ij}, \qquad and \qquad R = g^{ij} R_{ij}. \quad (14)$$

Also, it can be seen that the anti-symmetric part of the Ricci tensor satisfies [6]

$$R_{[ij]} = n\nabla_{[i} T_{j]}. \quad (15)$$

Pure and hybrid tensors in Kaehlerian Weyl spaces are defined similar to the definitions in Riemannian spaces. Let T be a tensor field of type $(0,2)$ on KW_n. If T satisfies $T(JX,JY) = T(X,Y)$ for any vector fields X , Y and any almost complex structure J on KW_n (in local coordinates $T_{sr} J^s_j J^r_i = T_{ji}$) then T is said to be a hybrid tensor with respect to j and i and if $T(JX,JY) = -T(X,Y)$ (in local coordinates $T_{sr} J^s_j J^r_i = -T_{ji}$), then T is said to be a pure tensor with respect to j and i in a Kaehlerian Weyl space [10].

Let

$$H_{ij} = \frac{1}{2} R_{ijkl} F^{kl}, \; M_{ij} = g_{ki} R^k_j, \quad (16)$$

where, $R_{ijkl} = g_{ih} R^h_{jkl}$, $R^k_j = R^h_{jkl} g^{kl}$. Then the following relations hold [10] :

$$a) \quad M_{ij} = \left(\frac{n-2}{n}\right) R_{ij} + \frac{2}{n} R_{ji} = R_{ij} + 2n(R_{ji} - R_{ij}) \quad (17)$$

$$b) \quad H_{ij} = -M_{hj} F_i^{\ h} = M_{ih} F_j^{\ h}, \quad (18)$$

$c)\ H_{hi}F_j^h = -H_{jh}F_i^h = M_{ji},$ (19)

$d)\ H_{hi}F^{hi} = -M_{hi}g^{hi} = -R,$ (20)

$e)\ R_{ijkl} + R_{jikl} = 4\nabla_{[k}T_{l]}g_{ij}.$ (21)

$f)\ H_{ij} + H_{ji} = 0.$ (22)

2. Generalized conformally recurrent Kaehlerian Weyl spaces

An n-dimensional Weyl space is called a generalized recurrent Weyl space if its curavture tensor R_{lijk} satisfies the condition

$$\dot{\nabla}_r R_{lijk} = A_r R_{lijk} + B_r(g_{lj}g_{ik} - g_{lk}g_{ij}), \quad (23)$$

where A and B are non-zero two 1-forms of weights 0 and -2, respectively, [11]. By putting $G_{lijk} = g_{lj}g_{ik} - g_{lk}g_{ij}$, (23) becomes

$$\dot{\nabla}_r R_{lijk} = A_r R_{lijk} + B_r G_{lijk}. \quad (24)$$

If the 1-form B is zero, then the Weyl space reduces to recurrent Weyl space [8].

Now, we introduce the following definitions on recurrent Kaehlerian Weyl spaces.

Definition 2.1. An n-dimensional $(n = 2m)$ Kaehlerian Weyl space is said to be a generalized recurrent if its curvature tensor R_{lijk} of weight 2 satisfies the condition

$$\dot{\nabla}_r R_{lijk} = A_r R_{lijk} + B_r G_{lijk}, \quad (25)$$

where A and B are two 1-forms of weights 0 and -2, respectively. Such a space is denoted by $GRKW_n$.

If the 1-form B is zero, then $GRKW_n$ is recurrent Kaehlerian Weyl space.

On the other hand, the conformal curvature tensor C^h_{ijk} of W_n is given by Miron [12]

$$C^h_{ijk} = R^h_{ijk} + \delta^h_k L_{ij} - \delta^h_j L_{ik} + L^h_k g_{ij} - L^h_j g_{ik} - 2\delta^h_i L_{[jk]}, \quad (26)$$

where

$$L_{ij} = -\frac{R_{ij}}{(n-2)} + \frac{2}{n(n-2)}R_{[ij]} + \frac{Rg_{ij}}{2(n-1)(n-2)}, \quad (27)$$

and

$$L^h_k = g^{lh}L_{lk}. \quad (28)$$

Considering (17), (27) becomes

$$L_{ij} = -\frac{1}{n-2}M_{ij} + \frac{R_{ji} - R_{ij}}{n(n-2)} + \frac{Rg_{ij}}{2(n-1)(n-2)}, \quad (29)$$

and

$$L_{ij} = -\frac{1}{(n-2)}M_{ij} + \frac{1}{(n-4)(n-2)}(M_{ji} - M_{ij}) \quad (30)$$
$$\frac{1}{2(n-1)(n-2)}R\,g_{ij}.$$

Also, from (15), (17), (27) and (30) we obtain

$$L_{[ij]} = -\frac{1}{n}R_{[ij]} = -\nabla_{[i}T_{j]} = -\frac{1}{2(n-4)}(M_{ij} - M_{ji}). \quad (31)$$

Definition 2.2. An n-dimensional $(n = 2m, m > 2)$ Kaehlerian Weyl space is said to be generalized conformally recurrent if its conformal curvature tensor C_{lijk} of weight 2 satisfies the condition

$$\dot{\nabla}_r C_{lijk} = A_r C_{lijk} + B_r G_{lijk} \quad (32)$$

where A, B are two 1-forms of weights 0 and -2, respectively and $C_{lijk} = C^h_{ijk}g_{hl}$.

We can state the following theorem concerning the generalized conformally recurrent Kaehlerian Weyl space.

Theorem 2.1. A Kaehlerian Weyl space KW_n is generalized conformally recurrent if and only if it is generalized recurrent.

Proof: Assume KW_n is generalized conformally recurrent. Transvecting (26) by g_{hl} we get

$$C_{lijk} = R_{lijk} + g_{kl}L_{ij} - g_{jl}L_{ik} + g_{ij}L_{lk} - g_{ik}L_{lj} - 2g_{il}L_{[jk]}. \quad (33)$$

By taking the prolonged covariant derivative of (33) and using (32), (33) we have,

$$\begin{aligned}&\dot{\nabla}_r R_{lijk} + g_{kl}\dot{\nabla}_r L_{ij} - g_{jl}\dot{\nabla}_r L_{ik} + g_{ij}\dot{\nabla}_r L_{lk}\\&-g_{ik}\dot{\nabla}_r L_{lj} - 2g_{il}\dot{\nabla}_r L_{[jk]}\\&= A_r[R_{lijk} + g_{kl}L_{ij} - g_{jl}L_{ik} + g_{ij}L_{lk}\\&-g_{ik}L_{lj} - 2g_{il}L_{[jk]}] + B_r G_{lijk}.\end{aligned} \tag{34}$$

Transvecting (34) by F^{jk} and using (16), (17), (18) and (30) we obtain

$$\begin{aligned}&\frac{(n-3)}{(n-2)}\dot{\nabla}_r H_{li} + \frac{1}{(n-2)}\dot{\nabla}_r H_{il}\\&-\frac{1}{(n-1)(n-2)}F_{li}\dot{\nabla}_r R + \frac{1}{(n-4)}g_{il}F^{jk}\dot{\nabla}_r M_{jk}\\&= A_r\left[\begin{aligned}&\frac{(n-3)}{(n-2)}H_{li} + \frac{1}{(n-2)}H_{il}\\&-\frac{1}{(n-1)(n-2)}RF_{li} + \frac{1}{(n-4)}g_{il}F^{jk}M_{jk}\end{aligned}\right]\\&+\frac{1}{2}B_r G_{lijk}F^{jk}.\end{aligned} \tag{35}$$

Also, transvecting (35) by F^{li} and using (20) we get

$$\dot{\nabla}_r R = A_r R + \frac{n(1-n)}{(n-2)}B_r. \tag{36}$$

On the other hand, multiplying (35) by g^{li} and using (18) we obtain

$$F^{jk}(\dot{\nabla}_r M_{jk}) = A_r F^{jk}M_{jk} + \frac{1}{2}B_r G_{lijk}F^{jk}g^{li}. \tag{37}$$

Since $G_{lijk}F^{jk}g^{li} = 0$

$$F^{jk}\dot{\nabla}_r M_{jk} = A_r F^{jk}M_{jk}. \tag{38}$$

Substituting (36) and (38) in (35) we get

$$\dot{\nabla}_r H_{li} = A_r H_{li} + \frac{n-1}{n-2}B_r F_{li}. \tag{39}$$

By multiplying (39) by F_j^i we obtain,

$$\dot{\nabla}_r M_{lj} = A_r M_{lj} - \frac{n-1}{n-2}B_r g_{lj}. \tag{40}$$

Substituting (36) and (40) into (30) gives us,

$$\dot{\nabla}_r L_{ij} = A_r L_{ij} + \frac{1}{2(n-2)}B_r g_{ij}. \tag{41}$$

Using (41), (34) reduces to

$$\dot{\nabla}_r R_{lijk} = A_r R_{lijk} + \frac{n-1}{n-2}B_r G_{lijk}. \tag{42}$$

Hence, the necessary part of the theorem is proved.

Conversely, assume that KW_n is generalized recurrent with 1-forms A and B, i.e:

$$\dot{\nabla}_r R_{lijk} = A_r R_{lijk} + B_r G_{lijk}. \tag{43}$$

Multiplying (43) by F^{jk} and using (16) we get

$$\dot{\nabla}_r H_{li} = A_r H_{li} + B_r F_{li}. \tag{44}$$

Transvecting (44) by F^{li} we obtain

$$\dot{\nabla}_r R = A_r R - nB_r. \tag{45}$$

Also, since $H_{li} = M_{lh}F_i^h$ and (44) we get

$$\dot{\nabla}_r M_{lj} = A_r M_{lj} - B_r g_{lj}. \tag{46}$$

Hence from (30), (44), (45), we get

$$\dot{\nabla}_r L_{ij} = A_r L_{ij} + \frac{1}{2(n-1)}B_r g_{ij}, \tag{47}$$

and

$$\dot{\nabla}_r L_{[ij]} = A_r L_{[ij]}. \tag{48}$$

Taking prolonged covariant derivative of (33) and using (47) and (48) we get,

$$\dot{\nabla}_r C_{lijk} = A_r C_{lijk} + \frac{(n-2)}{(n-1)}B_r G_{lijk}, \tag{49}$$

which implies that sufficiency part of the theorem is proved.

Remark: If, in particular, B_r is zero we obtain the results in [10].

3. Generalized conharmonically recurrent Kaehlerian Weyl spaces

Let $W_n(g_{ij}, T_k)$ and $\tilde{W}_n(\tilde{g}_{ij}, \tilde{T}_k)$ be two Weyl spaces with connections ∇_k and $\tilde{\nabla}_k$, respectively and let the map $\tau : W_n \to \tilde{W}_n$ be a conformal mapping. As a special case, let the transformed expressions of the fundamental metric tensor g_{ij}

and the coefficients of Weyl connection Γ^i_{kl} be in the following forms

$$\tilde{g}_{ij} = g_{ij}, \quad \tilde{g}^{ij} = g^{ij}, \qquad (50)$$
$$\tilde{\Gamma}^i_{kl} = \Gamma^i_{kl} + \delta^i_k P_l + \delta^i_l P_k - g_{kl} g^{im} P_m$$

where the vector P_k is called conformal mapping vector such that

$$P_k = T_k - \tilde{T}_k. \qquad (51)$$

Let A be the differentiable harmonic function with weight $\{p\}$ defined by

$$\overline{A} = e^{c\int P_j du^j} A, \quad c = \frac{2(1-p)-n}{2} \qquad (52)$$

and the vector field P_k satisfies [13]

$$g^{kl}\nabla_k P_l + \frac{1}{2}(n-2)P^k P_k = 0. \qquad (53)$$

If a conformal transformation with P_k satisfying (53) transforms a harmonic function into a harmonic function then it is called conharmonic transformation.

It is shown that the conharmonic curvature tensor K_{lijk} of $W_n(g_{ij}, T_k)$ can be given by [13]

$$K_{lijk} = C_{lijk} + \frac{R}{(n-2)(n-1)} G_{lijk}, \quad n > 2 \qquad (54)$$

where C_{lijk} is the conformal curvature tensor of Weyl space and $G_{lijk} = g_{lj} g_{ik} - g_{lk} g_{ij}$.

Definition 3.1. An n-dimensional ($n = 2m$) Kaehlerian Weyl space is said to be generalized conharmonically recurrent if its conharmonic curvature tensor K_{lijk} of weight 2 satisfies the condition

$$\dot{\nabla}_r K_{lijk} = A_r K_{lijk} + B_r G_{lijk}, \qquad (55)$$

where A and B are non-zero two 1-forms of weights 0 and -2, respectively.

Theorem 3.1. A Kaehlerian Weyl space is generalized recurrent if and only if it is generalized conharmonically recurrent.

Proof: Asume KW_n is generalized recurrent, then

$$\dot{\nabla}_r R_{lijk} = A_r R_{lijk} + B_r G_{lijk} \qquad (56)$$

From Theorem 2.1 and (49) we have

$$\dot{\nabla}_r C_{lijk} = A_r C_{lijk} + \frac{(n-2)}{(n-1)} B_r G_{lijk}. \qquad (57)$$

By taking the prolonged covariant derivative of (54) we get

$$\dot{\nabla}_r K_{lijk} = \dot{\nabla}_r C_{lijk} + \frac{1}{(n-1)(n-2)} G_{lijk} \dot{\nabla}_r R. \qquad (58)$$

Using (57) in (58) we obtain,

$$\dot{\nabla}_r K_{lijk} = A_r C_{lijk} + \frac{(n-2)}{(n-1)} B_r G_{lijk} + \frac{1}{(n-1)(n-2)} G_{lijk} \dot{\nabla}_r R. \qquad (59)$$

Considering (45), (59) becomes

$$\dot{\nabla}_r K_{lijk} = A_r C_{lijk} + \frac{(n-2)}{(n-1)} B_r G_{lijk} + \frac{1}{(n-1)(n-2)} G_{lijk} (A_r R - nB_r). \qquad (60)$$

Therefore, from (54) we get

$$\dot{\nabla}_r K_{lijk} = A_r K_{lijk} + \frac{(n-4)}{(n-1)} B_r G_{lijk}. \qquad (61)$$

Hence the necessary part of the theorem is proved.

Conversely, assume that

$$\dot{\nabla}_r K_{lijk} = A_r K_{lijk} + B_r G_{lijk}, \qquad (62)$$

then, (58) becomes

$$A_r K_{lijk} + B_r G_{lijk} = \dot{\nabla}_r C_{lijk} + \frac{1}{(n-1)(n-2)} G_{lijk} \dot{\nabla}_r R. \qquad (63)$$

Using (54), we have

$$A_r \left(C_{lijk} + \frac{R}{(n-1)(n-2)} G_{lijk} \right) + B_r G_{lijk} = \dot{\nabla}_r C_{lijk} + \frac{1}{(n-1)(n-2)} G_{lijk} \dot{\nabla}_r R. \qquad (64)$$

Multiplying both sides of (64) by F^{jk} and using (16), we get

$$2A_r\left[\begin{array}{l}\frac{(n-3)}{(n-2)}H_{li}+\frac{1}{(n-2)}H_{il}-\frac{1}{(n-1)(n-2)}RF_{li}\\+\frac{1}{(n-4)}g_{il}F^{jk}M_{jk}\end{array}\right] \quad (65)$$

$$+A_r\frac{R}{(n-1)(n-2)}G_{lijk}F^{jk}+B_rG_{lijk}F^{jk}$$

$$=2\left[\begin{array}{l}\frac{(n-3)}{(n-2)}\dot{\nabla}_rH_{li}+\frac{1}{(n-2)}\dot{\nabla}_rH_{il}-\frac{1}{(n-1)(n-2)}F_{li}\dot{\nabla}_rR\\+\frac{1}{(n-4)}g_{il}F^{jk}\dot{\nabla}_rM_{jk}\end{array}\right]$$

$$+\frac{1}{(n-1)(n-2)}G_{lijk}F^{jk}\dot{\nabla}_rR$$

Since $G_{lijk}F^{li}F^{jk}=2n$, by transvecting (20) with F^{li} and using (20) we obtain

$$\dot{\nabla}_rR=A_rR-\frac{n(n-2)}{(n-4)}B_r,\quad (n>4). \quad (66)$$

Hence, by using (55) and (58) we get

$$\dot{\nabla}_rC_{lijk}=A_rC_{lijk}+\frac{(n-2)^2}{(n-1)(n-4)}B_rG_{lijk},\quad (n>4). \quad (67)$$

From Theorem 2.1.

$$\dot{\nabla}_rR_{lijk}=A_rR_{lijk}+\frac{(n-1)}{(n-4)}B_rG_{lijk},\quad (n>4). \quad (68)$$

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References

[1] Scholz, E. (2008). Weyl Geometry in Late 20th Century Physics arXiv.org/math arXiv:1111.3220; *Beyond Einstein*, *Proceedings Mainz Conference September*, Bach V., Rowe D. (eds.). Einstein Studies. Basel: Birkhäuser.

[2] Santamato, E. (1984). Geometric Definition of the Schrödinger equation from classical mechanics in curved Weyl space. *Phys. Rev*. D *29*, 216-222.

[3] Scholz, E. (2011). Weyl geometric gravity and "breaking" of electroweak symmetry , to appear in Annalen der Physik, arXiv.org/hep-th arXiv: 1102. 3478.

[4] Calderbank, D. M. J. & Pedersen, H. (2000). Selfdual Spaces With Complex Structures, Einstein-Weyl Geometry And Geodesics. *Annales de l'institut Fourier*, *50*, 921-963 .

[5] Tiwari, S. D. (2000). Geometry of Quantum Theory: Weyl-Kaehler Space, *Proceedings of the International Conference on "Geometry, Analysis and Applications"*, Banaras Hindu University, Edited by R. S. Pathak, World Scientific. 129-138, arXiv.org/quant-ph arXiv:0109048.

[6] Norden, A. (1976). Affinely connected spaces, GRFML, Moscow, (in Russian).

[7] Hlavaty, V. (1949). Theorie d'immersion d'une W_m dans W_n . *Ann. Soc. Polon. Math*, *21*, 196-206.

[8] Canfes, E. Ö. & Özdeğer, A. (1997). Some applications of prolonged covariant differentiation in Weyl space. *Journal of Geometry*, *60*, 7-16.

[9] Demirbüker, H. & Özdemir, F. (1998). Almost Hermitian, Almost Kaehlerian and Almost Semi-Kaehlerian Structures in Weyl Spaces. *Buletinnul Sthntific Universitath Politehnica Din Timisoara Matematica-Fizica*, *43*(57), 1-7.

[10] Özdemir, F. & Yıldırım, G.Ç.(2005). On conformally recurrent Kaehlerian Weyl spaces. *Topology and its applications*, *153*, 477-484.

[11] Canfes, E. Ö (2006). On Generalized Recurrent Weyl Spaces and Wong's Conjecture. *Differential Geometry and Dynamical Systems*, *8*, 34-42.

[12] Miron, R. (1968). Mouvements confermes dans les espaces W_n et N_n , *Vol. 19*, Tensor, N. S.

[13] Özen, F. & Altay, S. (2000). On Totally Umbilical Hypersurface with Conharmonic Curvature Tensor, Steps in Diff. Geom. *Proceedings of the Colloqium*, 243-250.