
Estimation of depth and structural index (model type) from 2D magnetic data based on the multiples of amplitude of the analytic signal-A new technique

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Abstract

In this paper a new method is proposed for interpretation of 2D magnetic data, using multiples of the analytic signal method, in which the analytic signal of magnetic anomaly is used directly to compute the depth and the structural index of the source instead of using its higher order derivatives. This method only needs the computation of the first order derivatives of the magnetic anomaly, so the results are more stable than the results obtained by the other existing analytic signal methods. This method is applied on synthetic magnetic data with and without noise, and the proposed method can successfully obtain the depth and the structural index of the sources. We also applied this method to interpret a real magnetic data over a shallow source related to the SOURK Iron Ore mine in Iran, whose source parameters are known from closely core drilling data, and the estimated results are in agreement with the true values.

Keywords: Analytic signal; multiples; derivative; Iran

1. Introduction

Developing fully automated techniques for determining generalized source characteristics of magnetic anomalies is a goal of potential-field geophysicists. This has become particularly important recently because large volumes of magnetic data are being collected for environmental and geologic applications.

Interpretation of potential field data provides important information about geological sources which are not exposed. Many automated methods have been developed to estimate depth, width, magnetization, and horizontal position of magnetic sources. The most widely used of these methods are Werner deconvolution (Werner, 1953; Hartman et al., 1971; Jain, 1976; Hansen and Simmonds, 1993), Euler deconvolution (Thompson, 1982; Reid et al., 1990; Mushayandebvu et al., 2001; Nabighian and Hansen, 2001; Stavrev and Reid, 2007), and analytic signal (Nabighian, 1972, 1974, 1984; Roest et al., 1992; Bastani and Pedersen, 2001) techniques.

Nabighian (1972) showed that the horizontal and vertical derivatives of the magnetic anomaly produced by a 2D source form a Hilbert transform

pair and define an analytic signal. An important property of the 2D analytic signal is that its amplitude is the envelope of its underlying signal (Kanasewich, 1981)-the horizontal or vertical derivative in the 2D magnetic problem. It follows that the magnitude of the gradient of magnetic data (henceforth referred to as the total gradient) is equal to the envelope of both the horizontal and vertical derivatives over all possible inclinations. For processing magnetic data, the amplitude of the analytic signal in 2D is remarkable in that it allows one to obtain a signal that is independent of the source magnetization direction. Attempts have been made to generalize the analytic signal to 3D.

Initially, some authors (Nabighian, 1974; Atchuta Rao et al., 1981; Roest et al., 1992; MacLeod et al., 1993; Hsu et al., 1996) used the feature of the analytic signal to estimate the depth of the source, but the disadvantage of these methods is that their methods could only be applied to some particular geologic sources. To solve this disadvantage, some authors improved the analytic signal methods (Debeglia and Corpel, 1997; Smith et al., 1998; Hsu et al., 1998; Thurston et al., 1999, Salem and Ravat, 2003). The improved analytic signal methods can compute both the depth and the structural index of the source, but they require computing the third-order derivatives of the magnetic data, which can dramatically enhance the effect of noise. Recent

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developed analytic signal methods (Salem et al., 2004; Salem, 2005; Ma and Du, 2012) used the derivatives of analytic signal to compute the depth and the structural index, which only need to compute the second order derivatives of magnetic data, and can achieve more accurate results than the above methods. Li (2006) proved that the analytic signal in three dimensions is disturbed by the magnetization direction, so the analytic signal methods are hard to apply in the interpretation of gridded magnetic anomaly.

In general, for geological applications, the structural indices are assumed using some prior information and so some values of structural indices could be used. Fig. (1) shows a sketch of some common geologic models that are widely used in the application of Euler's deconvolution method.

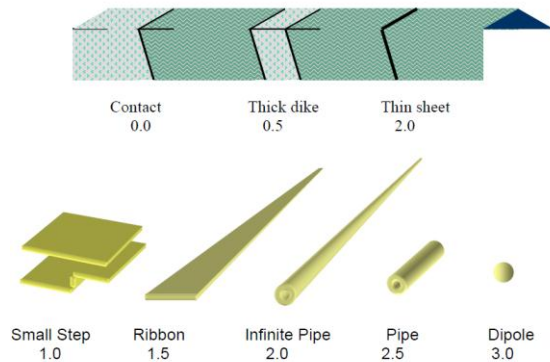


Fig. 1. Values of structural indexes of commonly used shapes in geology (Essam, 2005).

2. Analytic signal

The analytic signal $\mathbf{A}(x)$ of a potential field $\varphi(x)$ measured along the x-axis at a constant level caused by a 2D body striking along the y-axis can be written as a complex quantity (Nabighian, 1974),

$$\mathbf{A}(x) = \frac{\partial \varphi(x)}{\partial x} + i \frac{\partial \varphi(x)}{\partial z}, \quad (1)$$

where $\partial \varphi(x)/\partial x$ and $\partial \varphi(x)/\partial z$ are a Hilbert transform pair, and bold symbol represents vector, i is the imaginary number, z and x are Cartesian coordinates for the vertical direction and the direction perpendicular to strike.

The amplitude of the 2D analytic signal is (Nabighian, 1972; 1974; 1984):

$$|A(x)| = \sqrt{\left(\frac{\partial \varphi(x)}{\partial x}\right)^2 + \left(\frac{\partial \varphi(x)}{\partial z}\right)^2}, \quad (2)$$

The 2D analytic signal of the magnetic anomaly generated by the sources that are located at horizontal location $x=0$ and depth z are given in Table 1.

Table 1. Analytic signal of the 2D magnetic sources at depth z , and location $x=0$ (Doo, et al, 2007)

Model type	depth	location	equation
Magnetic contact	z	0	$ A(x, z) = \alpha \frac{1}{(x^2 + z^2)^{1/2}}$
Magnetic sheet (dyke)	z	0	$ A(x, z) = \alpha \frac{1}{(x^2 + z^2)}$
Cylinder	z	0	$ A(x, z) = \alpha \frac{1}{(x^2 + z^2)^{3/2}}$

where, α is a constant value and given by

$$\alpha = 2M \sin d(1 - \cos^2(I) \sin^2 A) \quad (3)$$

where M is magnetization, I is inclination of magnetization vector and A is direction of magnetization vector.

Based on the equations listed in Table 1, one can generalized the 2D response of the analytic signal due to magnetic sources located at horizontal location x_0 , depth z_0 can be given by:

$$|A(x, z)| = \frac{\alpha}{\left[(x - x_0)^2 + (z - z_0)^2 \right]^{(N+1)/2}} \quad (4)$$

where N is the structural index of the source, which represents the structural index of the source, $N=0$ for a contact, $N=1$ for a vertical dike, $N=2$ for a horizontal cylinder and $N=3$ for a dipole.

The maximum value of the analytic signal occurred directly above the center of the sources. Therefore, the horizontal location x_0 of the source can be determined by the peak of analytic signal, and the value of $A(x, z)$ at the horizontal location x_0 can be given by

$$A(x_0, 0) = \frac{\alpha}{(z_0^2)^{(N+1)/2}} \quad (5)$$

We can compute the amplitude of the analytic signal of our data set by equation 1 and the value of $A(x_0, 0)$ can be obtained from equation 5.

1.2. Multiples of analytic signal

It is fundamental to the proposed method to use multiples of the analytic signal (not its derivatives),

so that the analytic signal of the data is computed using equation 2, then its maximum value and also the horizontal location corresponding to it are determined using equation (5) (Ma and Li, 2012).

We can obtain different multiples of $A(x_0, 0)$, named it $rA(x_0, 0)$, and also the horizontal coordinate x_1 corresponding to r ($r < 1$) using an interpolation algorithm such as cubic spline. The value of r is selected by the interpreter (user). The horizontal locations of the analytic signal's multiples are shown as x_1 , x_2 and so on respectively. For example, term $A(x_1, 0)$ represents the amplitude of the analytic signal in horizontal location x_1 . In other words, the r multiple of the analytic signal in x_1 given by

$$A(x_1, 0) = rA(x_0, 0) = \frac{\alpha}{\left[(x_1 - x_0)^2 + z_0^2 \right]^{(N+1)/2}}. \quad (6)$$

We can also obtain the horizontal coordinate x_2 of the r^2 multiples of $A(x_0, 0)$, and the expression of $A(x, z)$ at the horizontal location x_2 can be given by

$$A(x_2, 0) = r^2 A(x_0, 0) = \frac{\alpha}{\left[(x_2 - x_0)^2 + z_0^2 \right]^{(N+1)/2}}. \quad (7)$$

For example, when $r=1/2$, $\frac{1}{2} A(x_0, 0)$ and $\frac{1}{4} A(x_0, 0)$ are determined using equation 6 and 7 respectively. Then we can estimate its horizontal location (x_1 and x_2) using a spline interpolation algorithm. The above definitions are shown in Fig. 2 schematically.

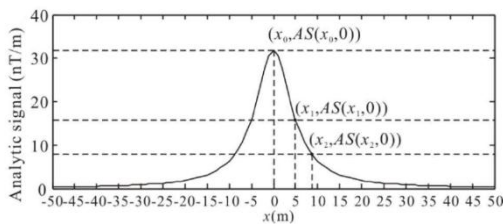


Fig. 2. The maximum value of the analytic signal and corresponding multiples (Ma and Li, 2012)

We can see that the equation 5 multiplied by equation 7 is equal to the square of equation 6, which can be written as

$$A(x_0, 0) \times A(x_2, 0) = [A(x_1, 0)]^2 \quad (8)$$

Substituting the equations 5, 7 and 6 into equation 8, we can obtain

$$\frac{\alpha}{(z_0^2)^{(N+1)/2}} \times \frac{\alpha}{\left[(x_2 - x_0)^2 + z_0^2 \right]^{(N+1)/2}} = \frac{\alpha^2}{\left[(x_1 - x_0)^2 + z_0^2 \right]^{N+1}} \quad (9)$$

Rearranging the equation

Eliminating the term α^2 from equation 9 we have:

$$\frac{1}{(z_0^2)^{(N+1)/2} \left[(x_2 - x_0)^2 + z_0^2 \right]^{(N+1)/2}} = \frac{1}{\left[(x_1 - x_0)^2 + z_0^2 \right]^{N+1}} \quad (10)$$

$$= z_0^2 \left[(x_2 - x_0)^2 + z_0^2 \right] = \left[(x_1 - x_0)^2 + z_0^2 \right]^2$$

Rearranging the above equation for z_0^2 we can obtain

$$z_0^2 = \frac{(x_1 - x_0)^4}{(x_2 - x_0)^2 - 2(x_1 - x_0)^2}. \quad (11)$$

The depth of the source can be estimated by the equation 11. After obtaining the depth of the source, we can use the ratio of equation 6 to equation 5 to compute the structural index of the source. We can obtain

$$N = 2 \times \frac{\log r}{\log \frac{z_0^2}{(x_1 - x_0)^2 + z_0^2}} - 1. \quad (12)$$

It's a way of determining which integer value is correct, and having discovered that, back-substitute the integer value determined this way. A good test would be the stability of the near-integer N solution as r varies. If N varies with r , and is not a near-integer, then the method is inapplicable. If N is constant and near-integer for a reasonable range of r , the method is valid for that anomaly and that source body.

3. Tests on synthetic magnetic anomalies

To test the performance of the proposed method, we apply it to composite magnetic anomaly of a thin vertical dike. The dike is located in the middle of the profile with its top depth of 5 m. The inclination and declination of induced magnetic field are 45° and 0° , respectively, and the sampling interval of the data is 1 m. Figure 3a shows the magnetic anomaly of the dike. Fourier transform was used to compute the horizontal and vertical derivatives of the magnetic anomaly. Figure 3b shows the analytic signal of the data in 3a. As can be seen from Fig. 3b, the horizontal location x_0 of

the source is 205 m, and the value of $A(x_0, 0)$ can also be obtained. In this example, the value of r is set to $1/2$. We can obtain the values of $1/2A(x_0, 0)$ and $1/4A(x_0, 0)$, and use cubic splines interpolation method to compute the corresponding horizontal coordinates. The horizontal coordinate x_1 of $1/2A(x_0, 0)$ is 209.5726 m, and the horizontal coordinate x_2 of $1/4A(x_0, 0)$ is 212.6755 m. We use equations 11 and 12 to compute the depth and the structural index of the source, respectively. The depth of the source is 5.0568 m, and the structural index of the source is 1.05. The inversion results show that the presented method can successfully obtain the depth and structural index of the causative source, but there is slight difference between the inversion results and the true values, a possible reason is that we use cubic splines interpolation method to obtain the horizontal coordinates rather than the theoretical coordinates.

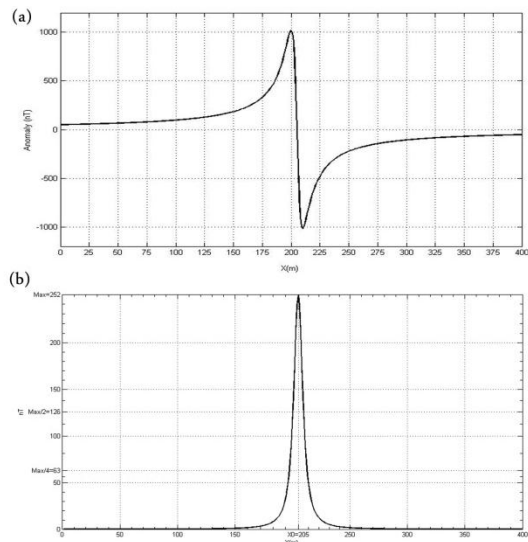


Fig. 3. (a) Noise-free magnetic anomaly generated by a dike model located at a horizontal location of 205 m and a top depth of 5 m. (b) Analytic signal of the anomaly in 1a. the maximum value (peak point) of the analytic signal located directly

In the interpretation of real data, the noise is an unavoidable element. Figure 4a shows the magnetic anomaly of a dike model corrupted by 8% Gaussian noise. The model is located at depth of 25m. The inclination and declination of the induced magnetic field are 60° and 10° , respectively, and the sampling interval of the data is 1 m. Figure 4b shows the AS of the data in Fig. 4a. As can be seen from Fig. 4b, the horizontal location x_0 of the source is 205. We still use the cubic spline interpolation method to compute the horizontal coordinate. The horizontal coordinate of $1/2A(x_0, 0)$ is 227.7404 m, and the horizontal coordinate of $1/4A(x_0, 0)$ is 243.3337 m.

Equations 11 and 12 are used to compute the depth and the structural index of the source, respectively. The depth of the source is 25.009 m, and the structural index of the source is 1.1503, the inversion results are very close to the real values used in forward modeling. It is noted that these results are obtained without applying any noise reducing techniques and certify the stability of this method. Fortunately, the noise level of modern aeromagnetic instruments has become very low, thus this method can provide reliable results from real magnetic data.

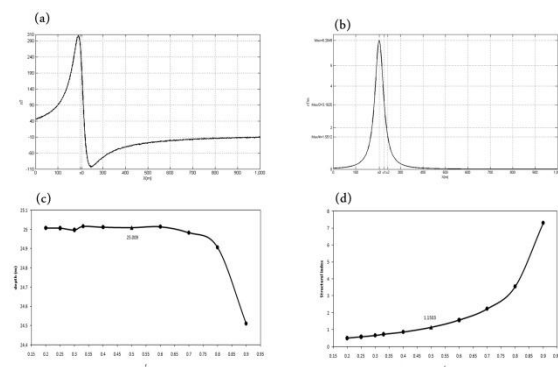


Fig. 4. (a) Magnetic anomaly generated by dyke in depth 25m corrupted by random noise with mean zero and mean square deviation of 1.5 nT. (b) Analytic signal of the data in 4a. (c) Estimated depth of the source by the equation 11 as a function of the value r . (f) Estimated structural index of the source by the equation 12 as a function of the value r

We also provided approaches to get more accurate results. This way is to use the mean values of the inversion results computed by a series of pair of points according to the change of r . Figure 4c shows the estimated depths as a function of r . Figure 4d shows the estimated structural index as a function of r . As can be seen from the results, the errors of the inversion results get smaller as the calculation points increase, so this strategy can improve the accuracy of inversion results to some extent.

We present another theoretical example from dyke model with high degree of noise level. Figure 5a shows the magnetic anomaly of a dike model corrupted by 20% Gaussian noise. The model is located at depth of 25m. The inclination and declination of induced magnetic field are 60° and 10° , respectively, and the sampling interval of the data is 1 m. Figure 5b shows the AS of the data in Fig. 5a. As can be seen from Figure 5b, the horizontal location x_0 of the source is 500. We use the cubic spline interpolation method to compute the horizontal coordinate. The horizontal coordinate of $1/2A(x_0, 0)$ is 521.7521 m, and the horizontal coordinate of $1/4A(x_0, 0)$ is 537.8522 m. We use the

equations 11 and 12 to compute the depth and the structural index of the source, respectively. The depth of the source is 28.3636 m, and the structural index of the source is 1.2729. It is noted that these results are obtained without applying any noise reducing techniques and certify the stability of this method. We also can use a noise reducing technique to get more accurate results, and the upward continuation filter is considered. We continued the data to an elevation of 1m. Figure 5c shows the AS of the filtered magnetic anomaly. Figure 5d shows the estimated depths as a function of r and Fig. 5e shows the estimated structural index as a function of r . We applied the presented method to the filtered data, and the best estimated depth and structural index are 25.8781 and 1.1317 with respect to $r=0.6$, respectively, so the precision of the inversion results are increased due to upward continuation.

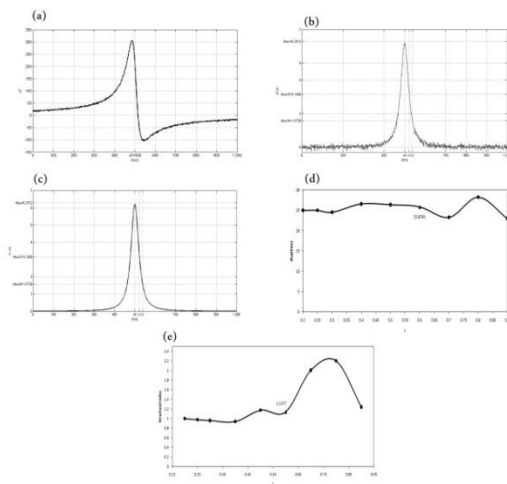


Fig. 5. (a) Magnetic anomaly generated by dyke in depth 25m corrupted by random noise with mean zero and mean square deviation of 8 nT. (b) Analytic signal of the data in 5a. (c) Analytic signal of the Filtered magnetic anomaly using upward continuation at 1m. (d) Estimated depth of the source by the equation 11 as a function of the value r . (e) Estimated structural index of the source by the equation 12 as a function of the value r

In the next example, we applied the present method on the magnetic anomaly of dyke with a lot of synthetic noise. In this case the dyke was located at depth of 15 m and corrupted with 40% random noise. Figure 6a shows the magnetic effect of the model. Figure 6b shows the analytic signal of the data in Fig. 6a. Figure 6c and 6d show the estimated depth and structural index as a function of r respectively. As seen, when the noise level of the data set is augmented the accuracy of the results decreased. However, in $r=0.5$ the results are closer to the real values with respect to others.

In many real data, the magnetic sources are not isolated and there is interference between them. We

apply the presented method to synthetic magnetic anomalies generated by 2 adjacent dikes located at 26 m depth. The horizontal distance of adjacent dikes is increasing and the other source parameters are the same.

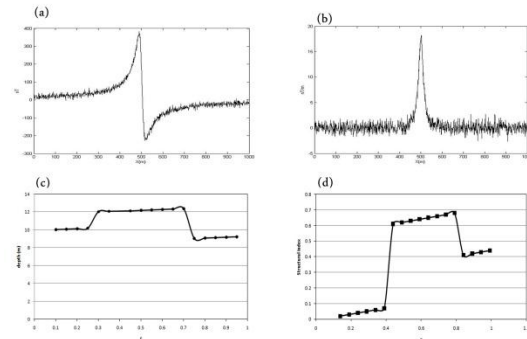


Fig. 6. (a) Magnetic anomaly generated by dyke in depth 15m corrupted by random noise with mean zero and mean square deviation of 100 nT. (b) Analytic signal of the data in 6a. (c) Estimated depth of the source by the equation 11 as a function of the value r . (d) Estimated structural index of the source by the equation 12 as a function of the value r

Figure 7a shows the magnetic anomalies of the dikes, and the horizontal locations of the dikes are 45 and 55 from left to the right. Figure 7b shows the analytic signal of the data in 6a. We apply the proposed method to interpret the magnetic data, when the value of r is increased. Figure 7c and 7d show the estimated depth and structural index of the source as a function of r . So there are big differences between inversion results and true values due to the interface effect. We deduce that the horizontal distance between dikes should be about 150 m or farther to achieve more reasonable results, it follows that the distance between dikes should be at least five times their depths.

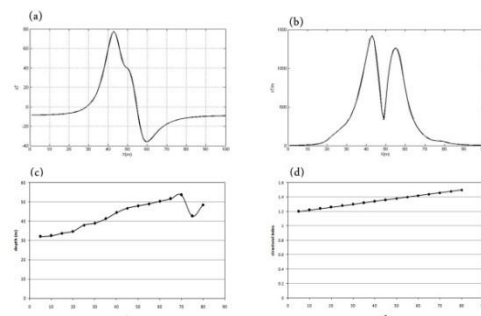


Fig. 7. (a). composite magnetic anomalies of 2 dykes models. Horizontal location of the dykes is 45m and 55m from left respectively. Right dyke is at depth 35 m and left dyke is at depth 40m. (b) analytic signal of the data in Fig.7a. (c) estimated depth of the dataset in a using present method as a function of r . (d) estimated structural index of the causative bodies as s function of r

For further evaluation of the present method it

was tested on synthetic magnetic data from horizontal cylinder with depth to top 40m and radius 35m. The inclination and declination of induced magnetic field are 90° and 10° , respectively, and the sampling interval of the data is 1 m. Figure 8a shows the magnetic anomaly of a horizontal cylinder model. Figure 8b shows the AS of the data in Figure 8a. Figure 8c and 8d show the estimated depth and structural index of the horizontal cylinder respectively.

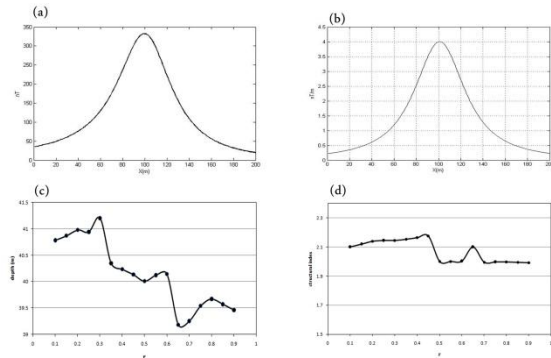


Fig. 8. (a) Magnetic anomaly generated by horizontal cylinder in depth 40m and radius 35m (b) Analytic signal of the data in 8a. (c) Estimated depth of the source by the equation 11 as a function of the value r . (d) Estimated structural index of the source by the equation 12 as a function of the value r .

4. Application to real magnetic data

The studied area is located in southeastern Iran in Yazd Province. This region is considered as Iron-bearing ore body (Sourk Iron ore mine). The total intensity magnetic dataset provided by the authors was recorded by Proton precision magnetometer with sensitivity at 0.001nT. The acquisition of magnetic data in the studied area was carried out along north-south profiles perpendicular to the main tectonic structures. The magnetic data was acquired to allow structural interpretations to be carried out in regions where little or no data had previously existed. Figure 9 shows the total field anomaly map covering the studied area. In the study area, the magnetic signature is generally smooth, with anomalies of different wavelengths and amplitudes, denoting the different magnetic contents of the causative bodies such as altered dykes.

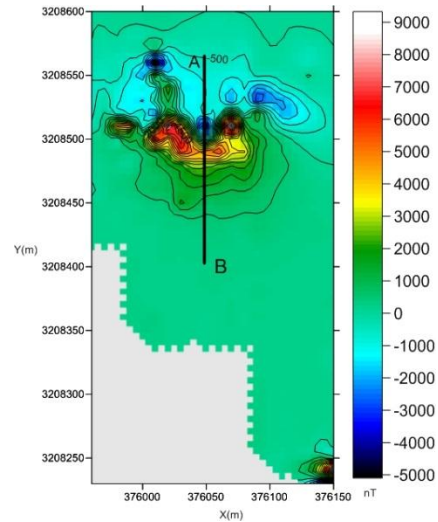


Fig. 9. Magnetic anomaly of the studied area in Yazd Province, Iran. The section "AB" used in new depth estimation method is presented in this paper

The magnetic profile shown in Fig. 10 is resampled in the North-South direction with an interval of 5 m. From closely-spaced drilling information, it is known that the depth to the top of the dike (from the surface) is about 30 m. Figure 10a shows the magnetic anomaly along section "AB". The analytic signal of the data in 10a is shown in Fig. 10b. The value of r is set to 1/2. According to Fig. 10b, we can ascertain that the horizontal coordinate of the source is 24.1 m, and the horizontal coordinates that correspond to the values of 1/2 and 1/4 maximum value are 27.4 m and 28.9 m, respectively.

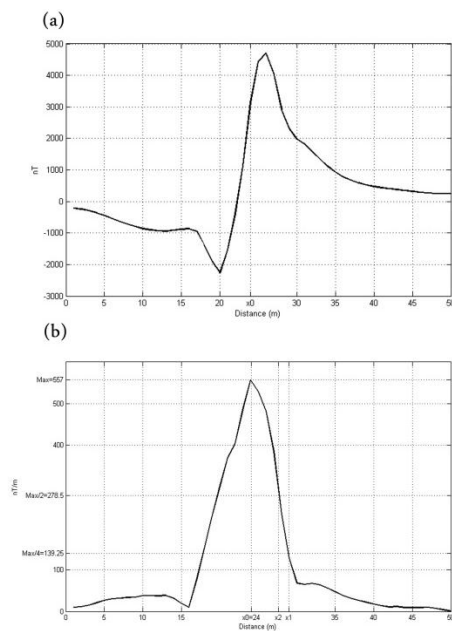


Fig. 10. (a) Magnetic anomaly along section "AB" from Fig. 7. (b) Analytic signal of the data in 8a

The estimated depth is 28.1053 m computed by the equation 11, which is close to the true depth. The estimated structural index is 1.1501 computed by the equation 12, approaching the value of a dike source (SI=1).

5. Conclusion

In this paper, a new method was presented to interpret magnetic anomalies, which used the multiples of the points on the analytic signal of the original magnetic anomaly to compute the depth and the structural index of the source. The important advantage of the proposed method is that this technique uses only the first order derivatives of dataset, and therefore is less sensitive to noise than techniques that use higher derivatives such as enhanced local wavenumber. The applicability of this method is demonstrated on synthetic magnetic data from dyke in different depths with and without random noise. In all cases, the new method can provide accurate results. In the case that the corrupting noise is high, use of a smoothing filter such as low-pass filter or upward continuation is suggested. The similar uncertainty is appearance because of the interference of adjacent sources, so the distance of the sources cannot be too small. We also test this method on real magnetic anomaly over an Iron Ore mine in Iran, and the results so obtained are in accordance with the real values.

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