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The generalization of structure factor for rods by polygon section in two-dimensional phononic crystals

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Abstract

The purpose of this paper is the generalization of structure factor for rods by polygon section in two dimensional phononic crystals. If we use the plane wave expansion method (PWE) for the propagation of acoustic waves in 2D phononic crystals, structure factor will be an important quantity. In order to confirm the obtained relations, we have calculated the band structure for XY and Z vibration modes in 2D phononic crystals and the propagation of bulk acoustic waves (BAW) are considered. In addition, the effect of sides' number on the band structure and the complete band gaps width are investigated. Phononic crystals studied in this paper are composites medium of a square lattice consisting of parallel nickel rods embedded in epoxy. The frequency is calculated by PWE in the condition of elastic rigidity to the solid inclusions. The results showed that, when the section of rods have 2n+2 (n is even) and 2n+1($n \in N$) by increasing sides number of the rod sections, the bands of XY mode shift to lower-frequency, the bands are smoother and the width of the band gap increases, but the band of Z mode has not changed by n variations. Moreover, when the section of rods have 2n+2 (n is odd) by increasing sides number of rod sections, the band structure of XY mode changes slightly and the width of the complete band gap is decreased. This confirms the effect of lattice symmetry on the complete band gap width. But, the band structure of Z mode has not changed.

Keywords*:* Band gap; phononic crystal; Polygon section; structure factor

1. Introduction

Elastic wave propagation in periodic structures has been the subject of intensive researches and it has received a great deal of attention during the past two decades [1-7]. These periodic structures, phononic crystals, are also called the elastic band gap (EBG) materials. These inhomogeneous elastic media are composed of one [8, 9], two [1, 10, 11], or three [12, 13] dimensional periodic arrays of inclusions embedded in a matrix. Due to their periodic structure, these materials may exhibit under certain conditions, absolute acoustic band gaps, i.e. forbidden bands which are independent of the propagation direction of the incident elastic wave. Because of this property, these structures have extensive practical applications, for instance, in the construction of sound shields and filters [14-19], in the refractive devices such as sound-wave focusing acoustic lenses [20, 21]. Also, they are used in the selective frequency waveguides [22]. In view of these applications, it is important to know how to design structures with the phononic gap as wide as possible. Besides the physical properties of the PC crystal component materials, the phononic gap width is found to depend strongly on the lattice symmetry, as well on the scatter shape [23, 24].

The phononic crystal solid/solid materials in two-dimensional are composed of periodic arrays of rods inclusions, under the assumption of wave propagation in the plane perpendicular to the axis rods. In this case, the vibration modes decouple in the mixed-polarization modes (XY) with the elastic displacement u perpendicular to the rod axes and in the transverse modes (Z) with u parallel to the inclusions. To reveal wide acoustic band gaps, we first require a large contrast in physical properties, such as density and speed of sound, between the inclusions and the matrix, next, a sufficient filling factor of inclusions [25].

In this paper, we generalize structure factor for rods by polygon section in two-dimensional phononic crystals. Also, the wave propagation of bulk acoustic is studied in square array of Nickel rods with polygon section embedded in epoxy. We reviewed formulation of plane wave method for elastic waves propagating in two-dimensional phononic crystal. Moreover, we calculate the band gap for XY and Z vibration modes, and the effect of sides number on the band structure and band gaps are investigated.

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2. Theory

The behavior of elastic waves propagating in a solid can be described by equation of motion, and can be obtained by considering the stresses acting on a volume element. Application of Newton's laws gives,

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sum_k \frac{\partial T_{ik}}{\partial r_k}, \quad (1)$$

Here ρ, u_i, T_{ik} and r_k are mass density, component of displacement vector, component of stress tensor and the position vector, respectively. The component of stress tensor is given as:

$$T_{ij} = \sum_{kl} c_{ijkl} u_{k,l}. \quad (2)$$

Here c_{ijkl} is elastic stiffness tensor. By considering the eigenfrequencies and normal mode, equation (1) may be written as [26],

$$\left[\rho\omega_k^2 \delta_{il} - c_{ijlk} k_k k_j\right] \boldsymbol{e}_l(k) = 0. \quad (3)$$

Here ω_k is eigenfrequency for wave vector k. By using Voiget notation and cubic symmetry, we have three elastic constants c_{11}, c_{12} and c_{44}. Due to the spatial periodicity of the phononic structure, the material constant $\rho(r)$ and $c_{ijlk}(r)$ are periodic functions of the position. It means that ρ and C are functions of the coordinates x and y where the z axis defines the direction of the rod axis. Then we can expand in the Fourier series with respect to two-dimensional reciprocal lattice vector

$$\rho(\mathbf{r}) = \sum_G \rho(\mathbf{G}) \exp(\mathrm{i}\mathbf{G}.\mathbf{r}), \quad (4)$$

and

$$C_{ij}(\mathbf{r}) = \sum_G C_{ij}(\mathbf{G}) \exp(\mathrm{i}\mathbf{G}.\mathbf{r}). \quad (5)$$

Where r is the position vector of components x and y and $\mathbf{G}$= (G_1, G_2) are the reciprocal lattice vectors in the xoy plane. The Fourier coefficients $\rho(\mathbf{G})$ and $C_{ij}(\mathbf{G})$ take the forms

$$\rho(\mathbf{G}) = \frac{1}{A} \iint d^2 r\, \rho(\mathbf{r}) \exp(-\mathrm{i}\mathbf{G}.\mathbf{r}), \quad (6)$$

and

$$C_{ij}(\mathbf{G}) = \frac{1}{A} \iint d^2 r\, C_{ij}(\mathbf{r}) \exp(-\mathrm{i}\mathbf{G}.\mathbf{r}). \quad (7)$$

The structure factor is defined as follows:

$$F_G = \frac{1}{A} \iint d^2 r \exp(-\mathrm{i}\mathbf{G}.\mathbf{r}), \quad (8)$$

here A is Unit cell area. If one of the denominators of F_G goes to zero, Eq. (4) and (5) will give the average density and elastic constant

$$\rho_G = \bar{\rho} = \rho^A f + \rho^B (1 - f), \quad (9)$$

and

$$C_{ij_G} = \overline{C_{ij}} = C_{ij}{}^A f + C_{ij}{}^B (1 - f). \quad (10)$$

Otherwise, Eq. (4) and (5) may be written as

$$\rho_G = (\rho^A - \rho^B) F_G \quad (11)$$

and

$$C_{ij_G} = \left(C_{ij}{}^A - C_{ij}{}^B\right) F_G. \quad (12)$$

Where f is filling fractions and mass density ρ^A and the elastic constants $C_{ij}{}^A$ are considered inside the rods and ρ^B and $C_{ij}{}^B$ in the background.

The structure factor for polygon section of filled inclusion will be as follows:

i) For polygons, in which the number of sides is 2n+1 (n ∈ N)

$$F_G = \frac{4\sin(rG_X sin2\theta)\sin(rG_y cos2\theta)}{a^2 G_X G_y} + \frac{1}{a^2 G_X} \sum_{i=1}^{n} \left\{ \frac{2}{(G_y - m_i G_X)} \left[-cos\left(G_y \sin\left(i\frac{\pi}{2} + (i-1)\theta\right) - G_X \cos\left(i\frac{\pi}{2} - (i-1)\theta\right)\right) + cos\left(G_y \cos(i+1)\left(\frac{\pi}{2} + \theta\right) - G_X sin(i+1)\left(\frac{\pi}{2} - \theta\right)\right) r \right] + \frac{2}{(G_y + m_i G_X)} \left[cos\left(G_y \sin\left(i\frac{\pi}{2} + (i-1)\theta\right) + G_X \cos\left(i\frac{\pi}{2} - (i-1)\theta\right)\right) r - cos\left(G_y \cos(i+1)\left(\frac{\pi}{2} + \theta\right) + G_X sin(i+1)\left(\frac{\pi}{2} - \theta\right)\right) r \right] \right\} \quad (13)$$

Where,

$$m_i = (-1)^{i-1} \frac{\cos\left[i\frac{\pi}{2} - (i-1)\theta\right] + \sin\left[(i+1)\left(\frac{\pi}{2} - \theta\right)\right]}{\sin\left[i\frac{\pi}{2} + (i-1)\theta\right] - \cos\left[(i+1)\left(\frac{\pi}{2} + \theta\right)\right]} \quad (14)$$

In the above sum, if i+1>n, it will take i+1=n.

ii) For polygons in which the number of sides is 2n+2 (n is even)

$$F_G = \frac{4\sin(rG_X sin2\theta)\sin(rG_y cos2\theta)}{a^2 G_X G_y} + \frac{1}{a^2 G_X} \sum_{i=1}^{n/2} \left\{ \frac{2}{(G_y - m_i G_X)} \left[-cos\left(G_y cos(i\pi + (i-1)\theta) - G_X sin(i\pi + (i-1)\theta)\right) r + cos\left(G_y cos((i+1)(\pi-\theta)) - G_X sin((i+1)(\pi-\theta))\right) r \right] + \frac{2}{(G_y + m_i G_X)} \left[cos\left(G_y cos(i\pi + (i-1)\theta) + G_X sin(i\pi + (i-1)\theta)\right) r - cos\left(G_y cos((i+1)(\pi-\theta)) + G_X sin((i+1)(\pi-\theta))\right) r \right] \right\} \quad (15)$$

Where,

$$m_i = (-1)^{i-1} \frac{\sin[i\pi + (i-1)\theta] + \sin[(i+1)(\pi-\theta)]}{\cos[i\pi + (i-1)\theta] - \cos[(i+1)(\pi-\theta)]}. \quad (16)$$

iii) For polygons in which the number of sides is 2n+2 (n is odd)

$$F_G = \frac{4\sin(rG_X sin2\theta)\sin(rG_y cos2\theta)}{a^2 G_X G_y} + \frac{1}{a^2 G_X}\sum_{i=1}^{(n-1)/2}\left\{\frac{2}{(G_y - m_i G_X)}\left[-cos\left(G_y cos\left((2i-1)(\frac{\pi}{2}+\theta)\right) - G_X sin\left((2i-1)(\frac{\pi}{2}+\theta)\right)\right)r + cos\left(G_y \sin\left((n-2i)(\frac{\pi}{2}+\theta)\right) - G_X \cos\left((n-2i)\left(\frac{\pi}{2}+\theta\right)\right)\right)r\right] + \frac{2}{(G_y + m_i G_X)}\left[cos\left(G_y sin\left(i\frac{\pi}{2} + (i-1)\theta\right) + G_X cos\left(i\frac{\pi}{2} + (i-1)\theta\right)\right)r - cos\left(G_y \sin\left((n-2i)(\frac{\pi}{2}+\theta)\right) + G_X \cos\left((n-2i)\left(\frac{\pi}{2}+\theta\right)\right)\right)r\right]\right\} \quad (17)$$

Where

$$m_i = \frac{\sin\left[(2i-1)(\frac{\pi}{2}+\theta)\right] + \cos\left[(n-2i)\left(\frac{\pi}{2}+\theta\right)\right]}{\cos\left[(2i-1)(\frac{\pi}{2}+\theta)\right] + \sin\left[(n-2i)\left(\frac{\pi}{2}+\theta\right)\right]}. \quad (18)$$

In the above sum, $\Sigma_{i=1}^{0}$ is equal to zero, where $\theta = 90^o - 180^o/n$ and r is radius of circumscribed circle of a polygon.

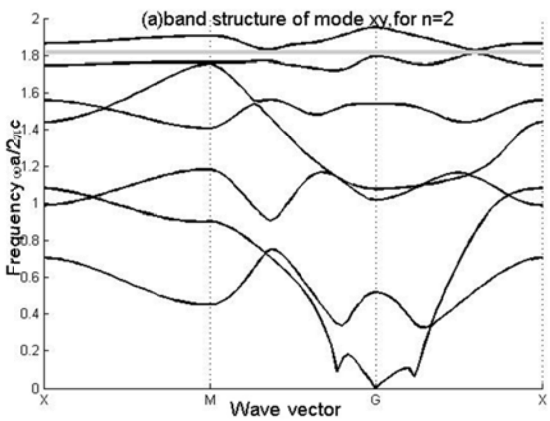
3. The method of calculation and results

Calculations were performed with the PWE method for the propagation of (BAW) for mixed polarization (longitudinal and shear horizontal) and shear vertical modes in 2D phononic crystals. These are composed of a 2D periodic square array of Ni rods in epoxy. To create the widest complete band gap, the ratio of the rod radius (the circumscribed circle radius of a polygon) to lattice constant varies from 0.05 to 0.5 and the best filling fraction was found to be about 0.4. The lattice parameter is 1mm. The rods made of Ni are assumed as an infinitely rigid solid. The choice of 1681 vectors of the reciprocal lattice for the computation ensures convergence of the eigenfrequency. The material parameter values in all the materials involved (Ni and epoxy) are specified in Table 1 [27].

Table 1. The elastic properties of the materials utilized in component material

Material	$\rho(kg/m^3)$	$C_{11}(N/m^2)10^{10}$	$C_{12}(N/m^2)\times 10^{10}$	$C_{44}(N/m^2)10^{10}$
Ni	8905	32.4	16.4	8
epoxy	1180	0.758	0.442	0.148

The band structure for the phononic crystals with polygon section 2n+2 sides (n=2 and n=12) is shown in Fig. 1. In Fig. 1(a) and (c), the dispersion relation is plotted for the propagation acoustic wave along X and Y with section 6 and 26 sides. Also, Fig. 1(b) and (d) are shown the propagation acoustic wave along Z for the same phononic crystal. By comparing Fig. 1(a) and (c) it can be seen that by increasing sides number of the rod sections, the band structure changes slightly. The bands shift to lower-frequency and are smoother. Furthermore, the width of the complete band gap increases. In Fig. 1(a), the complete band gap is formed between the sixth and seventh band but in Fig. 1(c), the complete band gap is formed between the fifth and sixth band. So the number of side effects on the location and width of the complete band gap can be predicted. By comparing Fig. 1(b) and (d), it can be seen that the band structure for the propagation acoustic wave along Z has not changed for n variations.



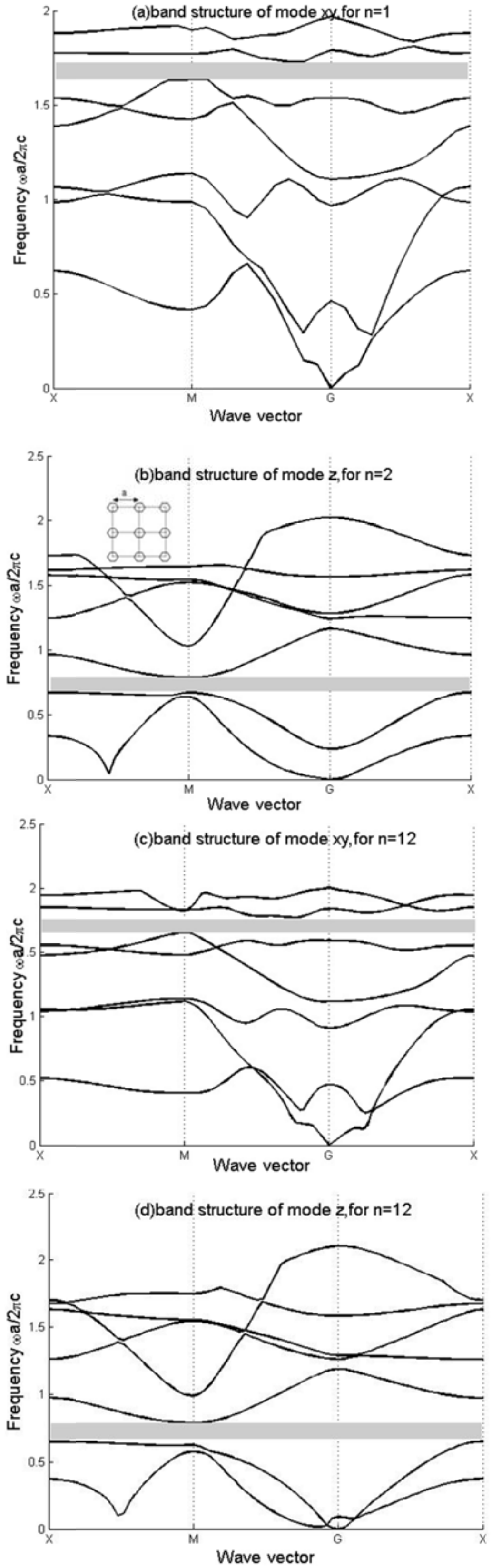
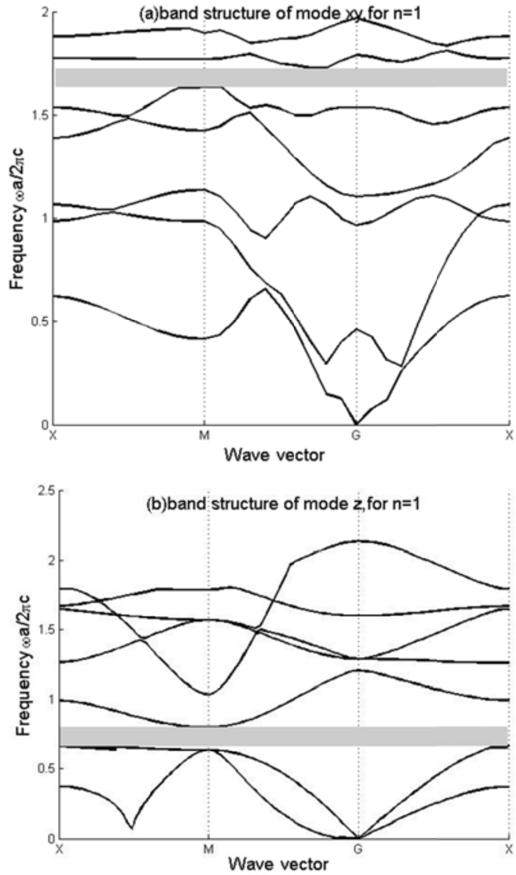


Fig. 1. The band structure in direction MXΓ for phononic crystals consisting of Ni rod in epoxy with 2n+2 sides (n is even) section, (a) n=2 and XY modes, (b) n=2 and Z mode, (c) n=12 and XY modes, (d) n=12 and Z mode

The band structure for phononic crystals with polygon section 2n+2 sides (n=1 and n=11) are shown in Fig. 1. In Fig. 2(a) and (c), the dispersion relation are plotted for the propagation acoustic wave along X and Y with section 4 and 24 sides. By comparing these figures, one can find that by increasing the number of sides, the band structure changes slightly and the width of the complete band gap is decreased, i.e. in this case, if rods sections are square, the widest complete band gap will be formed. This confirms the effect of lattice symmetry on the complete band gap width. Even in the previous case when n goes to infinite, the width of the band gap is less than the width in this case. Also, Fig. 1(b) and (d) show the propagation acoustic wave along Z for the same phononic crystal. By comparing these figures, one can see that the band structure for propagation acoustic wave along Z has not changed for n variations. The results of the band structure for the phononic crystals in which the section of rods have 2n+1 side are similar to the results of Fig. 1. These results corroborate others' results [30].



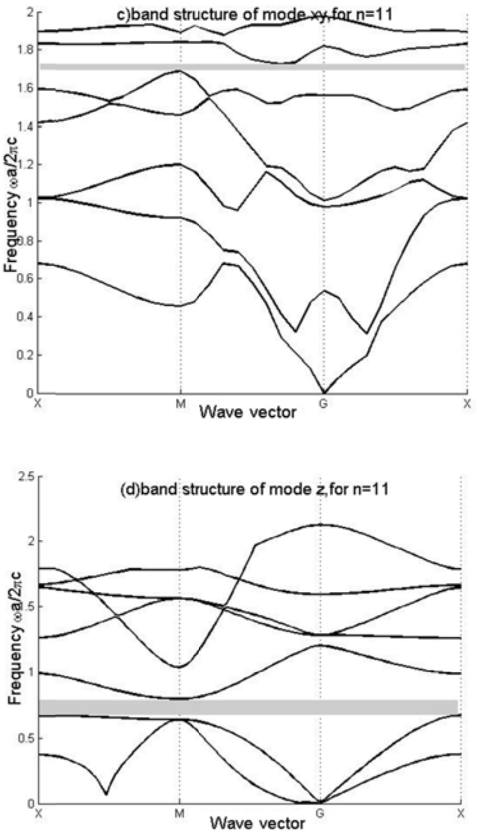


Fig. 2. The band structure in direction MXΓ for phononic crystals consisting of Ni rod in epoxy with 2n+2 sides (n is odd) section, (a) n=3 and XY modes, (b) n=3 and Z mode, (c) n=11 and XY modes, (d) n=11 and Z mode

4. Conclusion

In this paper, we generalized the structure factor for rods by polygon section in two dimensional phononic crystals. Then, the band gap for XY and Z vibration modes in two dimensional phononic crystals with square lattice were calculated. The propagation of bulk acoustic waves (BAW) was considered. In addition, the effect of sides' number on the band structure and the complete band gap width was investigated. The results showed that, when the section of rods have 2n+2(n is even) and 2n+1(n ∈ N) by increasing sides number of the rod sections, the bands of XY mode shift to lower-frequency, the bands are smoother, and the width of the band gap increases. So the number of side effects on location and width of the complete band gap can be predicted. However, the band of Z mode has not changed for n variations. Moreover, when the section of rods have 2n+2(n is odd) by increasing sides number of rod sections, the band structure of XY mode changes slightly and the width of the complete band gap is decreased. This confirms the effect of lattice symmetry on the complete band gap width. But, the band structure of Z mode has not changed.

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