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On fuzzy sub-hyperspaces of hypervector spaces

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Abstract

The purpose of this paper is the study of sub-hyperspaces of a hypervector space investigating some of their properties. In this regard, the notion of a normal fuzzy sub-hyperspace of hypervector space is introduced. Using it, we construct new fuzzy sub-hyperspaces. We also show that, under certain conditions, a fuzzy sub-hyperspace of a hypervector space is two-valued and takes the values 0 and 1.

***Keywords**:* Hypervector space; sub-hyperspace; fuzzy hypervector space; normal fuzzy sub-hyperspace

1. Introduction

Hyperstructure theory was born in 1934 when Marty [1] defined hypergroups, began to analyze their properties and applied them to groups, and rational algebraic functions. Now they are widely studied from a theoretical point of view and for their applications to many subjects of pure and applied properties [2]. In 1988, Maria Scafati Tallini [3] introduced the notion of hypervector space and studied their basic properties. In 1965, Zadeh [4] introduced the notion of a fuzzy subset of a non-empty set X as a function from X to $[0,1]$. Fuzzy set theory has been shown to be a useful tool to describe situations in which the data are imprecise or vague. Fuzzy sets handle such situations by attributing a degree to which a certain object belongs to a set. In 1971, Rosenfeld [5] applied this to the theory of grupoids and groups defining the concept of fuzzy group. Since then many papers have been published in the field of fuzzy algebra. One recent book [6] contains a wealth of applications. The concepts of fuzzy field and a fuzzy linear space over a field were introduced and discussed by Nanda [7]. In 1997, Katsaras and Liu [8] formulated and studied the notion of fuzzy vector subspaces over the field of real or complex numbers. Several characterizations on fuzzy vector spaces have been provided in [9-14]. Recently fuzzy set theory has been well developed in the context of hyperalgebraic structure theory [15-26] etc. In [15] Ameri introduced and studied the notion of fuzzy hypervector space over valued fields. In [16-19] Ameri et. al. studied more properties of fuzzy hypervector spaces as a generalization of fuzzy vector spaces. The purpose of this paper is the study of sub-hyperspaces of a hypervector space investigating some of their properties. Especially, some ways of constructing new fuzzy sub-hyperspaces from the old are considered. In this regard, we introduce the notion of a normal fuzzy sub-hyperspace of hypervector space. Using it, we construct new fuzzy sub-hyperspaces. We also show that, under certain conditions (such as: a non-constant normal fuzzy sub-hyperspace is maximal in the partial ordered set of anormal fuzzy sub-hyperspaces of a hypevector space), a fuzzy sub-hyperspace of a hypervector space is two-valued and takes the values 0 and 1.

2. Preliminaries

In this section, we present some definitions and simple properties of hypervector spaces and fuzzy subsets that we will use later.

Definition 2.1. *A map* $\circ : H \times H \to \mathcal{P}^*(H)$ *is called hyperoperation or join operation on the set* H*, where* H *is a non-empty set and* $\mathcal{P}^*(H) = \mathcal{P}(H) \setminus \{\varnothing\}$ *denotes the set of all non-empty subsets of* H.

Definition 2.2. *A hyperstructure is called a pair* $(H,\circ)$ *where* $\circ$ *is a hyperoperation on the set* H.

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If $x \in H$ and A, B are non-empty subsets of H, then

$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, A \circ x = A \circ \{x\}$, and $x \circ B = \{x\} \circ B$.

Definition 2.3. [27] Let K be a field and $(V,+)$ an Abelian group. A hypervector space over K is a quadruple $(V,+,\circ,K)$, where "$\circ$" is a mapping:

$$\circ : K \times V \to P^*(V)$$

such that for all $a,b \in K$ and $x, y \in V$ the following conditions hold:

1. $a \circ (x+y) \subseteq a \circ x + a \circ y,$
2. $(a+b) \circ x \subseteq a \circ x + b \circ x,$
3. $a \circ (b \circ x) = (ab) \circ x,$
4. $a \circ (-x) = (-a) \circ x = -(a \circ x),$
5. $x \in 1 \circ x$.

Remark 1. (i) In the right-hand side of the right distributive law (H1), the sum is meant in the sense of Frobenius, that is we consider the set of all sums of an element of $a \circ x$ with an element of $a \circ y$. Similarly, it is in the left distributive law (H2).
(ii) We say that $(V,+,\circ,K)$ is left distributive if
$\forall a,b \in K, \forall x \in V, (a+b) \circ x \supseteq a \circ x + b \circ x$
and strongly left distributive if
$\forall a,b \in K, \forall x \in V, (a+b) \circ x = a \circ x + b \circ x$.
In a similar way, we define the right distributive and strongly right distributive hypervector spaces, respectively. The hypervector space $(V,+,\circ,K)$ is called strongly distributive if it is both strongly left and strongly right distributive.
(iii) The left-hand side of the associative law (H3) means the set-theoretical union of all the sets $a \circ y$, where y runs over the set $b \circ x$, that is,

$$a \circ (b \circ x) = \bigcup_{y \in b \circ x} a \circ y.$$

(iv) Let $\Omega_V = 0 \circ 0$, where 0 is the zero of $(V,+)$. In [27], it is shown that if V is either strongly right or strongly left distributive, then Ω is a subgroup of $(V,+)$.

Several examples and other characterizations of hypervector spaces can be found in [18], [22], [28-37].

Througout the paper, unless otherwise specified, we assume that V is a hypervector space over the field K.

Definition 2.4. A non-empty subset W of V is called a sub-hyperspace if W is itself a hypervector space with the hyperoperation on V, that is,
$W \neq \varnothing$,
$\forall x, y \in W \Rightarrow x - y \in W,$
$\forall a \in K, \forall x \in W \Rightarrow a \circ x \subseteq W$.

In this case, we write $W \leq V$.
Let X be an ordinary set. By a *fuzzy set* μ in X, we mean a function $\mu : X \to [0,1]$ with the grade of membership $\mu(x)$ for $x \in X$. If $t \in [0,1]$, then $\mu_t = \{x \in X : \mu(x) \geq t\}$ is called a *level subset* of μ. We define $FS(X)$ the set of fuzzy sets of X. If $\mu \in FS(X)$ and $A \subseteq X$, then by $\mu(A)$, we mean
$\mu(A) = \bigwedge_{a \in A} \mu(a)$.

3. Fuzzy normal sub-hyperspaces

Definition 3.1. Let K be a field and $\nu \in FS(K)$. Suppose the following conditions hold:

1. $\nu(a+b) \geq \nu(a) \wedge \nu(b)$, $\forall a,b \in K$,
2. $\nu(-a) \geq \nu(a)$, $\forall a \in K$,
3. $\nu(ab) \geq \nu(a) \wedge \nu(b)$, $\forall a,b \in K$,
4. $\nu(a^{-1}) \geq \nu(a)$, $\forall a \in K \setminus \{0\}$.

Then, call ν *a fuzzy field in* K *and denote it by* ν_K.

Definition 3.2. [15] Let V be a hypervectorspace over a field K and ν a fuzzy field of K. A fuzzy set μ of V is said to be a fuzzy hypervector space of V over the fuzzy field ν_K, if for all $x, y \in V$ and all $a \in K$, the following conditions are satisfied:

1. $\mu(x+y) \geq \mu(x) \wedge \mu(y)$,
2. $\mu(-x) \geq \mu(x)$,
3. $\bigwedge_{y \in a \circ x} \mu(y) \geq \nu(a) \wedge \mu(x)$,
4. $\nu(1) \geq \mu(0)$.

If we consider $\nu = \chi_K$ the characteristic function of K, then μ is called a *fuzzy sub-hyperspace* of V.

Clearly, if μ is a fuzzy sub-hyperspace of V, then $\mu(0) \geq \mu(x)$ for all $x \in V$. Also, μ is a fuzzy sub-hyperspace of V if and only if μ_t is a sub-hyperspace of V for all $t \in [0,1]$.

Lemma 3.3. If μ is a fuzzy sub-hyperspace of V, then the set $V_\mu = \{x \in V \mid \mu(x) = \mu(0)\}$ is a sub-hyperspace of V.

Proof: Let $x, y \in V_\mu$. Then $\mu(x) = \mu(y) = \mu(0)$. Since μ is a fuzzy sub-hyperspace, it follows that $\mu(x-y) \geq \mu(x) \wedge \mu(y) = \mu(0) \wedge \mu(0) = \mu(0)$.

On the other hand, $\mu(x-y) \leq \mu(0)$. Hence we have $\mu(x-x) = \mu(0)$ and so $x - y \in V_\mu$. Also, for any $x \in V_\mu$ and $a \in F$, we get $\bigwedge_{y \in a \circ x} \mu(y) \geq \mu(x) = \mu(0)$. On the other hand, $\bigwedge_{y \in a \circ x} \mu(y) \leq \mu(0)$. Hence, we obtain $\bigwedge_{y \in a \circ x} \mu(y) = \mu(0)$, which shows that $a \circ x \subseteq V_\mu$. Consequently, the set V_μ is a sub-hyperspace of V.

Definition 3.4. A fuzzy sub-hyperspace μ of V is said to be normal if there exists $x \in V$ such that $\mu(x) = 1$. Note that if a fuzzy sub-hyperspace of V is normal, then $\mu(0) = 1$. Hence μ is a normal fuzzy sub-hyperspace if and only if $\mu(0) = 1$.

Theorem 3.5. Let μ be a fuzzy sub-hyperspace of V and let $\tilde{\mu}$ be a fuzzy set in V defined by $\tilde{\mu}(x) = \mu(x) + 1 - \mu(0)$ for all $x \in V$. Then $\tilde{\mu}$ is a normal fuzzy sub-hyperspace of V containing μ.

Proof: Let $x, y \in V$ and $a \in K$. Then

$$\tilde{\mu}(x-y) = \mu(x-y) + 1 - \mu(0) \geq (\mu(x) \wedge \mu(y)) + 1 - \mu(0) =$$
$$= (\mu(x) + 1 - \mu(0)) \wedge (\mu(y) + 1 - \mu(0)) = \tilde{\mu} \wedge \tilde{\mu}(y).$$

Also, we have

$$\bigwedge_{z \in a \circ x} \tilde{\mu}(z) = \bigwedge_{z \in a \circ x} \mu(z) + 1 - \mu(0) \geq \mu(x) + 1 - \mu(0) = \tilde{\mu}(x).$$

Clearly, $\tilde{\mu}(0) = 1$ and $\mu \subseteq \tilde{\mu}$. This completes the proof.

Corollary 3.6. If μ is a fuzzy sub-hyperspace of V satisfying $\tilde{\mu}(x) = 0$ for some $x \in V$, then $\mu(x) = 0$.

Lemma 3.7. Let χ_W be the characteristic function of a subset $W \subseteq V$. Then W is a sub-hyperspace of V if and only if χ_W is a fuzzy sub-hyperspace of V.

Proof: It follows from the discussion after Definition 3.2.

Theorem 3.8. For any sub-hyperspace W of V, the characteristic function χ_W is a normal fuzzy sub-hyperspace of V and $V_{\chi_W} = W$.

Proof: Straightforward.

Theorem 3.9. A fuzzy sub-hyperspace μ of V is normal if and only if $\tilde{\mu} = \mu$.

Proof: If $\tilde{\mu} = \mu$, then it is obvious that μ is a normal fuzzy sub-hyperspace of V. Assume that μ is a normal fuzzy sub-hyperspace of V and let $x \in V$. Then $\tilde{\mu}(x) = \mu(x) + 1 - \mu(0) = \mu(x)$, and hence $\tilde{\mu} = \mu$.

Theorem 3.10. If μ is a fuzzy sub-hyperspace of V, then $\widetilde{(\tilde{\mu})} = \tilde{\mu}$.

Proof: Straightforward.

Theorem 3.11. Let μ be a fuzzy sub-hyperspace of V. If there exists a fuzzy sub-hyperspace ν of V satisfying $\tilde{\nu} \subseteq \mu$, then μ is a normal fuzzy sub-hyperspace of V.

Proof: Suppose there exists a fuzzy sub-hyperspace ν of V such that $\tilde{\nu} \subseteq \mu$. Then $1 = \tilde{\nu}(0) \leq \mu(0)$, and therefore $\mu(0) = 1$.

Corollary 3.12. Let μ be a fuzzy sub-hyperspace of V. If there exists a fuzzy sub-hyperspace ν of V satisfying $\tilde{\nu} \subseteq \mu$, then $\tilde{\mu} = \mu$.

Proof: It is immediately obtained from Theorem 3.11 and definition of $\tilde{\mu}$.

Theorem 3.13. Let μ be a fuzzy sub-hyperspace of V and $f:[0,\mu(0)]\to[0,1]$ be an increasing map. Define a fuzzy set $\mu':V\to[0,1]$ by $\mu'(x)=f(\mu(x))$ for all $x\in V$. Then μ' is a fuzzy sub-hyperspace of V. In particular, if $f(t)\geq t$ for all $t\in[0,\mu(0)]$, then $\mu\subseteq\mu'$.

Proof: Let $x,y\in V$. Then

$$\mu'(x-y)=f(\mu(x-y))\geq f(\mu(x)\wedge\mu(y))=$$
$$=f(\mu(x))\wedge f(\mu(y))=\mu'(x)\wedge\mu'(y).$$

Also, if $a\in K$ and $x\in V$, then $\bigwedge_{z\in a\circ x}\mu'(z)=f(\bigwedge_{z\in a\circ x}\mu(z))\geq f(\mu(x))=\mu'(x)$.
Hence μ' is a fuzzy sub-hyperspace of V. Assume that $f(t)\geq t$ for all $t\in[0,\mu(0)]$. Then $\mu'(x)=f(\mu(x))\geq\mu(x)$ for all $x\in V$, which means $\mu\subseteq\mu'$.

Theorem 3.14. Let μ be a non-constant normal fuzzy sub-hyperspace of V, which is maximal in the partial ordered set of normal fuzzy sub-hyperspaces of V under fuzzy sets inclusion. Then μ is a two-valued fuzzy sub-hyperspace and takes the values 0 and 1.

Proof: We know that $\mu(0)=1$. Let $x\in V$ such that $\mu(x)\neq 1$. It is enough to show that $\mu(x)=0$. Assume that there exists $x'\in V$ such that $0<\mu(x')<1$. Define a fuzzy set $\nu:V\to[0,1]$ by $\nu(x)=\frac{1}{2}(\mu(x)+\mu(x'))$ for all $x\in V$. Then clearly ν is well-defined. Let $x,y\in V$. Then

$$\nu(x-y)=\frac{1}{2}(\mu(x-y)+\mu(x'))\geq\frac{1}{2}((\mu(x)\wedge\mu(y)))+\mu(x'))=$$
$$=(\frac{1}{2}(\nu(x)+\mu(x')))\wedge(\frac{1}{2}(\mu(y)+\mu(x')))=\nu(x)\wedge\nu(y).$$

Also, if $a\in K$ and $x\in V$, then

$$\bigwedge_{z\in a\circ x}\nu(z)=\frac{1}{2}(\bigwedge_{z\in a\circ x}\mu(z)+\mu(x'))\geq\frac{1}{2}(\mu(x)+\mu(x'))=\nu(x)\cdot$$

Hence ν is a fuzzy sub-hyperspace of V. Now we have

$$\tilde{\nu}(x)=\nu(x)+1-\nu(0)=$$
$$=\frac{1}{2}(\mu(x)+\mu(x'))+1-\frac{1}{2}(\mu(0)+\mu(x'))=\frac{1}{2}(\mu(x)+1).$$

So $\tilde{\nu}(0)=\frac{1}{2}(\mu(0)+1)=1$. Thus $\tilde{\nu}$ is a normal fuzzy sub-hyperspace of V. Also $\tilde{\nu}(0)=1>\tilde{\nu}(x')=\frac{1}{2}(\mu(x')+1)>\mu(x')$. We know that $\tilde{\nu}$ is non-constant. So by $\tilde{\nu}(x')>\mu(x')$ it follows that μ is not maximal, which is a contradiction. Therefore μ takes only values 0 and 1.

Theorem 3.15. Let μ be a fuzzy sub-hyperspace of V and let $\overline{\mu}$ be a fuzzy set in V defined by $\overline{\mu}(x)=\mu(x)\mu(0)$ for all $x\in V$. Then $\overline{\mu}$ is a normal fuzzy sub-hyperspace of V containing μ.

Proof: For any $x,y\in V$, we have

$$\mu(x-y)=\mu(x-y)\mu(0)\geq(1\mu(0))(\mu(x)\wedge\mu(y))=$$
$$=(\mu(x)\mu(0))\wedge(\mu(y)\mu(0))=\overline{\mu}(x)\wedge\overline{\mu}(y).$$

Also, if $a\in K$ and $x\in V$, we get

$$\bigwedge_{z\in a\circ x}\overline{\mu}(z)=(\bigwedge_{z\in a\circ x}\mu(z)\mu(0))\geq(\mu(x)\mu(0))=\overline{\mu}(x)\cdot$$

Hence $\overline{\mu}$ is a fuzzy sub-hyperspace of V. Clearly $\overline{\mu}(0)=1$ and $\mu\subseteq\overline{\mu}$.

Corollary 3.16. If μ is a fuzzy sub-hyperspace of V satisfying $\overline{\mu}(x)=0$ for some $x\in V$, then $\mu(x)=0$.

Proof: It is obvious.

Theorem 3.17. Let μ be a non-constant fuzzy sub-hyperspace of V such that $\overline{\mu}$ is maximal in the partial ordered set of normal fuzzy sub-hyperspace of V under fuzzy set inclusion. Then

1. μ is normal.
2. μ takes only the values 0 and 1.
3. $\chi_{V_\mu}=\mu$.
4. V_μ is a maximal sub-hyperspace of V.

Proof: Since μ is non-constant, $\tilde{\mu}$ is non-constant maximal. Also, $\tilde{\mu}$ is normal, which implies $\tilde{\mu}$ takes only the values 0 and 1 by Theorem 3.14. If $\tilde{\mu}(x)=1$, then $\mu(x)=\mu(0)$ and if $\tilde{\mu}(x)=0$, then $\mu(x)=\mu(0)-1$. By Corollary 3.6, we have $\mu(x)=0$ which implies $\mu(0)=1$. Therefore μ is normal, and also $\tilde{\mu}=\mu$ by Theorem 3.9, which proves (1) and (2).

(3) Clearly $\chi_{V_\mu} \subseteq \mu$ and χ_{V_μ} takes only the values 0 and 1. Let $x \in V$ and $\mu(x)=0$, then $\mu \subseteq \chi_{V_\mu}$. If $\mu(x)=1$, then $x \in V_\mu$ and so $\chi_{V_\mu}(x)=1$. In any case $\mu \subseteq \chi_{V_\mu}$.

(4) Since μ is non-constant, V_μ is a proper sub-hyperspace of V. Let W be a sub-hyperspace of V such that $V_\mu \subseteq W$. Then we obtain $\mu=\chi_{V_\mu} \subseteq \chi_W$. Since μ and χ_W are normal and $\mu=\tilde{\mu}$ is maximal in the partial ordered set of normal fuzzy sub-hyperspaces under fuzzy sets inclusion, we have $\mu=\chi_W$ or $\chi_W(x)=1$ for all $x \in V$, so $W=V$. If $\mu=\chi_W$, then $V_\mu = V_{\chi_W} = W$ by Theorem 3.8. Therefore V_μ is a maximal sub-hyperspace of V.

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